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Parametric Approach for Estimation of Technical Efficiency

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Abstract : In this paper an attempt is made to explain the basic concept of efficiency, frontier production function, technical efficiency, deterministic frontier, and stochastic frontier. Efficiency of a firm/industry refers to its performance in the utilization of resources at its disposal and is a relative concept. Technical efficiency of a production function is defined as the maximum quantity of output obtainable from given set of inputs. A failure to produce the greatest possible output means the technical decision is in efficient. Technical inefficiency can be obtained by the methods stochastic and deterministic production frontier models. The method discussed in this paper has several possible extension and generalization. Technical efficiency has many policy implications in various functional areas of modern management.

Keywords: Production function, Frontier, Efficiency, Technical Efficiency,

1. Introduction

Technical efficiency is the effectiveness with which a given set of inputs is used to produce an output. A firm is said to be technically efficient if a firm is producing the maximum output from the minimum quantity of inputs, such as labor, capital and technology. Technical efficiency means that natural resources are transformed into goods and services without waste. Charnes, Cooper and Rhodes extended Farrell's idea and proposed a model that generalizes the single-input, single-output ratio measure of efficiency of a single Decision-Making Unit (DMU) in multiple-inputs, multiple outputs setting. A DMU is an entity that produces outputs and uses up inputs. The technical efficiency of a DMU is computed using the engineering-like efficiency measure of efficiency as ratio of virtual output produced to virtual input consumed:

$$\text{Technical Efficiency} = \frac{\sum \text{Weighted output}}{\sum \text{Weighted input}}$$

Farrell (1957) assumed that observed input-per-unit-of-output values for firms would be above the unit isoquant. Figure 1.1 depicts the situation in which firms use two inputs of production, X_1 and X_2 to produce their output , such that the points, defined by the input-per-unit-of-output ratios,(X_1/Y, X_2/Y), are above the curve. The unit isoquant defines the input-per-unit-of-output ratios associated with the most efficient use of the inputs to produce the output involved. The deviation of observed input-per-unit-of-output ratios from the unit isoquant was considered to be associated with technical inefficiency of the firms involved. The ratio, OB/OA, is defined to be the technical efficiency of the firm with input-per-unit-of-output values at point A.

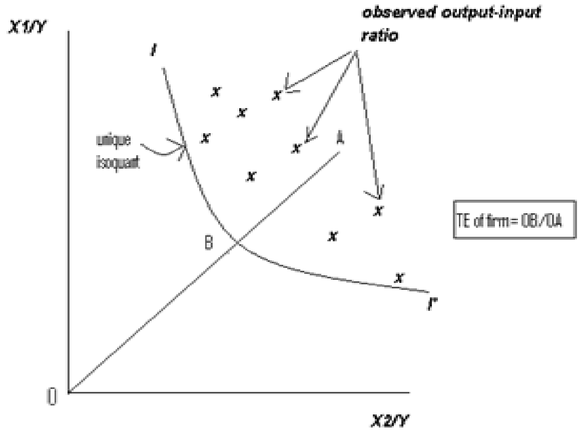

Fig 1.1. Technical Efficiency of firms in relative input space

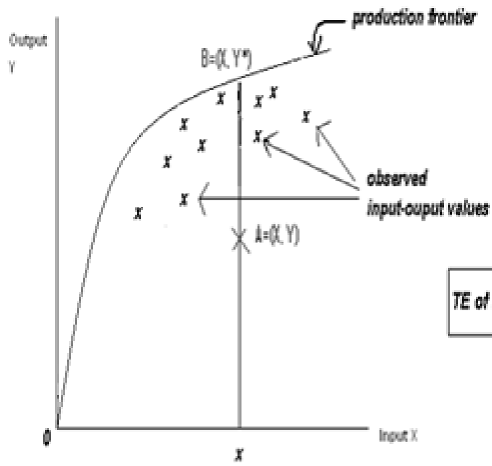

Fig 1.2. Technical Efficiency of firms in relative input space

A more general presentation of Farrell's concept of the production function (or *frontier)* is depicted in Figure 1.2

involving the original input and output values. The horizontal axis represents the inputs, X, associated with producing the output, Y. The observed input-output values are *below* the production frontier, given that firms do not attain the maximum output possible for the inputs involved, given the technology available. A measure of the technical efficiency of the firm which produces output, y, with inputs, x, denoted by point A, is given by y/y', where y" is the "frontier output" associated with the level of inputs, x (see point B). This is an *input-specific* measure of technical efficiency which is more formerly defined in the next section.

2. Production Function

The theoretical definition of production function expressing the maximum amount of output obtainable from given input bundle with fixed technology has been accepted for many decades. And for almost as long, econometricians have been estimating average production function. It has only been since the pioneering work of Farrell (1957) that serious consideration has been given to the possibility of estimating so called Frontier Production Function, in an effort to bridge the gap between theory and empirical work.

The standard definition of a production function is that it gives the maximum possible output for a given set of inputs; the production function therefore defines a boundary or a frontier. All the production units on the frontier will be full efficient. Technical efficiency can be modeled using either the deterministic or the stochastic production frontier. In the case of the deterministic frontier model the entire shortfall of observed output from maximum feasible output is attributed to technical inefficiency, whereas the stochastic frontier model includes the effect of random shocks to the production frontier.

There are two alternative approaches to estimate frontier model namely; Non-parametric approach which includes Data Envelopment Analysis and Parametric approach which includes Deterministic Frontier Approach and Stochastic Frontier Approach

A production frontier model can be written as

$$y_i = f(x_i, \beta) TE_i$$

(2.1)

Where y_i is the output of producer i (i = 1,2……..n), x_i is a vector of m inputs used by producer i , $f(x_i, \beta)$ is the production frontier and β is a vector of technology parameters to be estimated. Let TE_i be the technical efficiency of producer i,

$$TE_i = \frac{y_i}{f(x_i, \beta)}$$

(2.2)

Which defines technical efficiency as the ratio of observed output y_i to maximum feasible output $f(x_i, \beta)$. In the case $TE_i = 1$, y_i achieves its maximum feasible output of $f(x_i, \beta)$. If $TE_i < 1$, it measures technical inefficiency in the sense that observed output is below the maximum feasible output $f(x_i, \beta)$ is attributed to technical inefficiency

3. Methodology

There are two parametric approaches to find technical efficiency; namely Deterministic frontier model and Stochastic frontier model.

Deterministic frontier model: The first approach examined was the construction of the deterministic statistical frontier using statistical technique, such that all deviation from this frontier are assumed to be the result of inefficiency. That is, no allowance is made for noise or measurement error. In the primal from, the ability to incorporate multiple outputs is difficult. Considering the Cobb-Douglus production function

$$y_i = A x_1^{\beta_1} x_2^{\beta_2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . . x_i^{\beta_i} \varepsilon_i, \quad \varepsilon_i > 0 \qquad (3.1)$$

$$\ln y_i = \ln A + \beta_1 ln x_1 + \beta_2 ln x_2 + \dots\dots\dots\dots + \beta_i ln x_i + ln \varepsilon_i$$

$$ln y_i = x_i \beta - u_i$$

(3.2)

x_i's in logs and include a constant u_i is non-negative random variable. β is the vector parameters of the production frontier to be estimated and y_i represent the possible production level for the i-th sample firm

$$y_i = \exp(x_i \beta) \exp(-u_i)$$

(3.3)

$$y_i = f(x_i : \beta) \exp(-u_i)$$

(3.4)

Where $f(x_i : \beta)$ is a suitable function of the vector, u_i is a non negative random variable associated with firm specific factors which contribute to the i^{th} firm not attaining maximum efficiency of production.

The presence of the non negative random variable, u_i in the model defines the nature of technical inefficiency of the firm and implies that the random variable $\exp(-u_i)$ has values between 0 and 1. Thus it follows that the possible production y_i, is bounded above by the non- stochastic quantity $f(x_i{:}\beta)$. Hence the model (1) is referred to as a deterministic frontier production function. The inequality relationship

$$y_i \leq f(x_i{:}\beta), \qquad i = 1,2.\ldots\ldots\ldots n \tag{3.5}$$

The technical efficiency of a given firm is defined to be the factor by which the level of production for the firm is less than its frontier output. Given the deterministic frontier model (3.4), the frontier output for the i^th firm is $y_i^* = f(x_i{:}\beta)$ and so the technical efficiency for the i^th firm, denoted by TE_i is

$$TE_i = \frac{y_i}{y_i^*} = \frac{f(x_i{:}\beta)\exp(-u_i)}{f(x_i{:}\beta)}$$

$$TE_i = \exp(-u_i) \tag{3.6}$$

Stochastic Frontier Model: The second parametric approach namely the stochastic frontier, removes some of the limitations of the deterministic frontier. Its biggest advantage lies in the fact that it introduces a disturbance term representing noise, measurement error and exogenous shocks beyond the control of the production unit. This in turn permits the decomposition of deviation from the efficient frontier into two components inefficient and noise. However, in common with the deterministic approach an assumption regarding the distribution of this noise must be made along with those required for the inefficiency term and the production technology.

The stochastic frontier production function is defined by

$$y_i = f(x_i{:}\beta)\exp(v_i - u_i) \tag{3.7}$$

$i = 1,2.\ldots\ldots n$

Here the error term is composed by the measurement error (u_i) and random error (v_i), subject to $\varepsilon_i = v_i - u_i$. Hence if we assume that $u_i = 0$.

Technical efficiency of an individual firm is defined in terms of the corresponding frontier output, given the levels of inputs used by that firm.

Thus $$TE_i = \frac{y_i}{y_i^*} = \frac{f(x_i{:}\beta)\exp(v_i - u_i)}{f(x_i{:}\beta)\exp(-u_i)} = \exp(-u_i)$$

The basic structure of the stochastic frontier model (3.7) is depicted in figure 3 in which the productive activities of two firms, represented by i and j are considered. Firm i use inputs with value given by x_i and obtain the output y_i, but the frontier output y_i^* exceeds the value on the deterministic production function $f(x_i{:}\beta)$, because its productive activity is associated with favorable conditions for which the random error v_i is positive. However firm j uses inputs with values given by x_j and obtains the output y_j which has corresponding frontier output y_j^* which is less than the value on the deterministic production function $f(x_j{:}\beta)$ because its productive activity is associated with unfavorable conditions for which the random error v_j is negative.

In both cases the observed production values are less than the corresponding frontier values, but the (unobservable) frontier production values would lie around the deterministic production function associated with the firms involved.

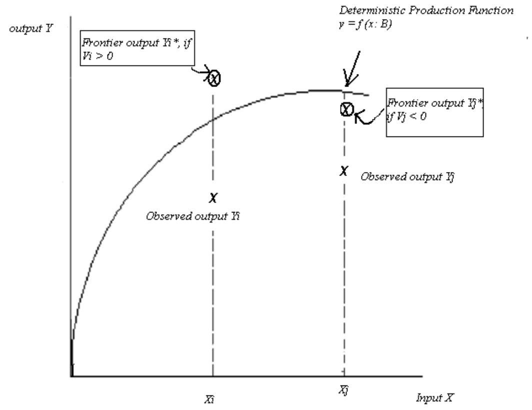

Fig 3: Stochastic Frontier Production Function

Considering the above figure it is evident that the TE of firm j is greater under the stochastic frontier model than for the deterministic frontier. That is firm j is judge technically more efficient relative to the maximum associated with the value of the deterministic function $f(x_j{:}\beta)$. Further firm i is judged technically less efficient relative to its favorable conditions than if its production is judged relative to the maximum associated with the value of the deterministic function $f(x_i{:}\beta)$.

4. Estimation

However for any given set of data the estimated technical efficiencies obtained by fitting a deterministic frontier will be less than those obtained by fitting a stochastic frontier, because the deterministic frontier will be estimated so that no output values will exceed it.

For the estimation of such model, the Corrected ordinary least squares (COLS)/Moments estimator is considered. Except for the constant term, the Ordinary least square (OLS) is unbiased and consistent. The bias of the constant term is the mean of (v-u), can be expressed as follows:

$$E(v-u) = -E(u)$$

$$-E(u) = -\frac{\sqrt{2}}{\pi}\sigma_u$$

Variances c an be estimated by

$$\sigma_u^2 = \left[\sqrt{2}/\pi\{\pi/(\pi-4)\}\mu'_3\right]^{2/3}$$

$$\sigma_v^2 = \mu'_2 - \{(\pi-2)/\pi\}\sigma_u^2$$

Where μ'_3 and μ'_2 are estimates of the third and second order moments of the OLS residuals. For this purpose the COLS/Moments estimator has been suggested, which involves correcting the estimate of the OLS intercept, by adding to the OLS estimated term the negative of the estimated bias, i.e. $\frac{\sqrt{2}}{\pi}\sigma_u$. Thus, the COLS/Moments estimator of all parameters except the intercept is the same as the OLS estimators. The element of σ_u is used to convert the OLS estimate of the constant term into the COLS/Moments estimate. If the third moment of least square residual is positive, σ_u^2 will be negative and there is no need to proceed beyond the OLS.

The stochastic production frontier model estimates the shape and placement of the frontier and estimates the average extent of technical efficiency over the observations included in the estimation.

5. Summary and Conclusion

Efficiency of a firm is measured either with respect to normatively desired performance of a firm/industry or with that of any other firm/industry. Thus the efficiency measures are basically the methods of comparing the observed performance of the firm/industry with some specified performance.

The production function is a purely technical relation which connects factor inputs and output(s). It describes the laws of production i.e. the transformation of factor inputs into product (outputs) at any particular time period. The production function represents the technology of the firm of an industry. The word frontier represents the concept of optimality. As the production frontier represents the maximum obtainable output for any given set of inputs, any deviation of a firm from the frontier is taken to indicate the extend of firms inability to produce maximum output from its given set of inputs and hence represents the degree of technical inefficiency. If the average technical efficiency of any production unit is low then there is a scope for raising the production through increase in production efficiency.

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