

# Vibration Analysis Of Laminated Composite Plates Using Design Of Experimentsapproach

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## **Abstract**

*This paper discusses the use of D-optimal designs and response surface methodology in predicting the natural frequency of laminated composite plate under clamped boundary condition. The finite element analyses have been carried out using D-optimal design in the design of experiments technique by varying the fibre orientations,  $-45^{\circ}$ ,  $0^{\circ}$ ,  $45^{\circ}$  and  $90^{\circ}$ . The natural frequencies were found by analyses which were done by finite element (FE) analysis software. A second order model has been established between the ply angles and natural frequency. The predicted values and measured values are fairly close, which indicates that the developed model can be effectively used to predict the natural frequency of laminated composite plates. The predicted values are confirmed by using validation experiments. The adequacy of the developed model is verified by using the coefficient of determination and residual analysis. The effect of side-to-thickness ratios, aspect ratios and modulus ratios are studied on the natural frequency for optimal stacking sequence.*

**Key words:** *Vibration analysis, finite element method, D-optimal designs, Response surface methodology, composite plate, residual analysis.*

## **1. Introduction**

Composites are being increasingly used in aerospace, marine and civil infrastructure owing to the many advantages they offer: high strength/stiffness for lower weight, superior fatigue response characteristics, facility to vary fibre orientation, material and stacking pattern, resistance to electrochemical corrosion, and other superior material properties. Usually, laminated composite plates fabricated from unidirectional plies of giving thickness and with fiber orientations limited to a small set of angles, e.g.,  $0^{\circ}$ ,  $45^{\circ}$ ,  $-45^{\circ}$  and  $90^{\circ}$  [1-5]. To avoid the resonant behavior of the laminated structures, the results of the free vibration analysis for the laminated composite structures in the structural design are very important. Also, the

composite structures whether used in civil, marine or aerospace are subjected to dynamic loads during their

operation. Therefore, there exists a need for assessing the natural frequency [6]. Therefore, for assessing the natural frequency of the laminated composite plate structures, the accurate mathematical model is required. However, free vibration analysis of laminated composite plates and shells has not received ample attention. Finite element method is especially versatile and efficient for the analysis of complex structural behaviour of the composite laminated structures. In the past, the structural behavior of plates and shells using the finite element method has been studied by a variety of approaches. Pandit et al [7] used a nine-noded isoparametric plate-bending element for the analysis of free undamped vibration of isotropic and fiber reinforced laminated composite plates. Ihsan Küçükrendeci and Ömer K. Morgül [8] investigated Effect of elastic boundary conditions on the linear undamped free vibrations of a five layer symmetrical laminated rectangular plate. The free vibration analysis of composite laminated right triangular plates is analyzed based on Mindlin plate theory by using the finite element method [9]. He studied the effects of the fiber orientation, boundary conditions, material properties, and geometric ratio  $a/h$  on the vibration frequencies. Khoa and Thinh [10] developed a rectangular non-conforming element based on Reddy's higher order shear deformation plate theory to analyze the laminated composite plates. They concluded that, the size of the mesh and the convergence of the method is involved by thickness ratio ( $h/a$ ). Rakesh Kumar *et al.* [11] employed a  $C^0$  isoparametric finite element formulation based on a shear deformable model of higher-order theory using a higher order facet shell element to study the free vibration analysis of composite and sandwich laminates. They also studied the parametric effects of degree of orthotropy, length to-thickness ratio, plate aspect ratio, number of layers and fibre orientation on the frequency and mode shapes. Patel *et al.* [12] was used

the finite element method for analyzing the free vibration of laminated anisotropic composite conical cylindrical shell structures. A simple two-noded shear flexible axi-symmetric shell element based on field consistency approach is employed. Reddy *et al.* [3] used the finite element method and ANN to optimize the stacking sequence of laminated composite plates using Distance based optimal design in the design of experiments.

In the present study, 16ply symmetric laminated composite plate has been modeled using linear layered structural shell element (SHELL 99). The composite plate is analysed using finite element analysis software for various fibre orientations in each layer i.e.,  $-45^0$ ,  $0^0$ ,  $45^0$  and  $90^0$ . The *D*-optimal design in the design of experiments technique has been employed to carry out the finite element analyses. A predictive model for natural frequency in terms of fibre orientations is then developed using response surface methodology. The developed model is tested with the test data which were never used for developing the model. The adequacy of the developed model is verified by using the coefficient of determination and residual analysis. The effect of side-to-thickness ratios, aspect ratios and modulus ratios are studied on the natural frequency for optimal stacking sequence.

## 2. Material and methodology

The plate material used in the present investigation is carbon fiber reinforced plastic (CFRP). The length and width of the plate are 250mm and thickness of the plate is 2.5mm. The ply orientation is treated as a design variable. The properties of CFRP are presented in Table 1.

Table 1: Material properties (carbon/epoxy)

$E_1(GPa)$	$E_2(GPa)$	$G_{12}(GPa)$	$G_{23}(GPa)$	$\nu_{12}$
128	11	4.48	1.53	0.25

There are many element types, in ANSYS software, available to model layered composite materials. In our FE analysis, shell 99 element is used. It is designed to model thin to moderately thick plate and shell structures with a side-to-thickness ratio of roughly 10 or greater. The linear layered structural shell element allows a total of 250 uniform-thickness layers. Alternatively, the element allows 125 layers with thicknesses that may vary bi-linearly over the area of the layer. It also has an option to offset the nodes to the top or bottom surface. The geometry of the shell 99 element is shown in Fig 1[13].

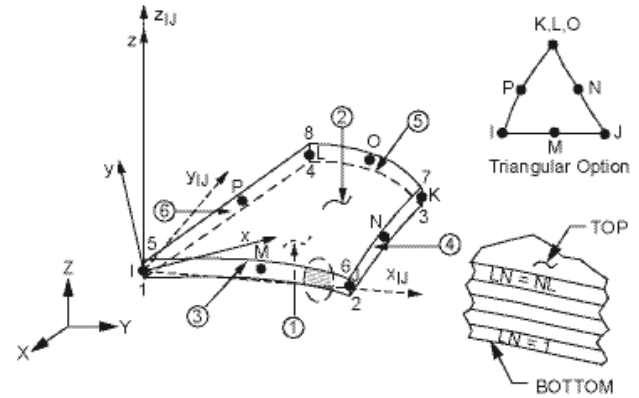


Fig 1. Geometry of 8-node element with six degrees of freedom

Where

$x_{ij}$  = Element x-axis if ESYS (Element co-ordinate system) is not supplied.

$x$  = Element x-axis if ESYS is supplied.

LN = Layer Number

NL = Total Number of Layers

## 2.2. Response surface methodology

### 2.2.1 Curve fitting

The Response surface methodology (RSM) is a collection of mathematical and statistical techniques that are useful for modeling, analysis and optimizing the process in which response of interest is influenced by several variables and the objective is to optimize this response. Response Surface Methodology uses quantitative data from appropriate experiments to determine and simultaneously solve multi-variable equation. The response surface methodology comprises regression surface fitting to obtain approximate responses, design of experiments to obtain minimum variances of the responses and optimizations using the approximated responses.

The response model is generally written as:

$$Y = f(x_1, x_2, \dots, x_n) + \epsilon \quad (1)$$

Where 'Y' is the response and  $\{x_1, x_2, \dots, x_n\}$  are independent (predictor) variables and  $\epsilon$  is an error term. Usually  $\epsilon$  includes effects such as measurement error in response, background noise, the effect of the other variables and so on. Usually  $\epsilon$  is treated as statistical error, often assuming it to have a normal distribution with mean zero and variance  $\sigma^2$ . The function  $f$  is normally selected to be polynomial. Usually a second order model is utilized in RSM and is written as

$$Y = \beta_0 + \sum_{j=1}^k \beta_j X_j + \sum_{j=1}^k \beta_{jj} X_j^2 + \sum_{i < j} \sum_{j=2}^k \beta_{ij} X_i X_j \quad (2)$$

The  $\beta$  coefficients, used in the above model can be calculated by means of using least squares technique [14].

**2.2.2. Design of experiments**

Design of Experiments (DOE) is a mathematical methodology that defines an optimal set of experiments in the design space, in order to obtain the most relevant information possible with the highest accuracy at the lowest cost. This scientific exploration of the design space replaces a tedious, manual, trial-and-error process, and is the fastest way to acquire the most relevant information with minimum computational effort. Traditional experimental designs (Full Factorial Designs, Fractional Factorial Designs, and Response Surface Designs) are appropriate for calibrating linear models in experimental settings where factors are relatively unconstrained in the region of interest. In some cases, however, models are necessarily nonlinear. In other cases, certain treatments (combinations of factor levels) may be expensive or infeasible to measure. D-optimal designs are model-specific designs that address these limitations of traditional designs.

The D-optimality criterion states that the best set of points in the experiment maximizes the determinant  $|X^T X|$ . "D" stands for the determinant of the X matrix associated with the model. A D-optimal design is generated by an iterative search algorithm and seeks to minimize the covariance of the parameter estimates for a specified model. This is equivalent to maximizing the determinant  $D = |X^T X|$ , where X is the design matrix of model terms (the columns) evaluated at specific treatments in the design space (the rows). Unlike traditional designs, D-optimal designs do not require orthogonal design matrices, and as a result, parameter estimates may be correlated. Parameter estimates may also be locally, but not globally, D-optimal. The D-optimal design uses the row-exchange and Co-ordinate exchange algorithms to generate the optimal designs.

A related measure of the moment matrix  $(X^T X/k)$  is the D-efficiency and can be calculated by using the following expression [14]:

$$D - efficiency = 100 \left( \frac{1}{N_D} |X^T X|^{1/p} \right)$$

(3)

Where  $N_D$  is the number of points in the design and  $p$  is the number of effects in the model including the intercept. If all variables are normalized so that they vary from -1 to 1, then the maximum value of the  $D_{eff}$  is 1. Furthermore, the quality of the set points can then be measured by *def*.

**3. Experimental Details**

**3.1. Validation shell 99 element- case study**

In order to validate the usage of the shell 99 element, numerical examples are solved for free vibration analysis.

Example 1: Free vibration analysis of simply supported  $[0^0/90^0/0^0/90^0/0^0]_s$  laminated composite square plate.

Consider a ten-layer simply supported laminated square plate with the following stacking sequences  $[0^0/90^0/0^0/90^0/0^0]_s$  and a side length of 0.2 m with a thickness of 0.00269 m. The material is considered to be T300/934 CFRP with the mechanical properties as:

$$E_1=120 \text{ GPa}, E_2=7.9 \text{ GPa}, G_{12}=5.5 \text{ GPa}, \rho=1580 \text{ kg/m}^3, \nu_{12} = 0.33$$

The natural frequency obtained by ANSYS is compared with the available literature [15, 16 and 17] and presented in Table 2. The convergence study is also presented in Table 2. The results using a free mesh show an excellent correlation to the results available in literature 15, 16 and 17].

Table 2. The natural frequency of simply supported  $[0^0/90^0/0^0/90^0/0^0]_s$  laminated composite square plate.

Mesh size	Natural frequency (Hz)
4x4	305.86
10x10	302.05
15x15	302.05
20x20	302.05
25x25	302.05
30x30	302.05
35x35	302.05
40x40	302.05
Super element [24]	303.116

**Example 2:** free vibration analysis of rectangular laminated composite plate with clamped all edges.

To verify the applicability of software for free vibration analysis, finite element results are compared with the

reference solution taken from literature [18]. The properties of the material are listed in Table 1. The plate parameters are as follows:

a = 250 mm; b = 500 mm; t= 1.04 mm;

The layer stacking sequence of the plate is  $[0/\pm 45/90]_s$ .

The play thickness considered as 0.13 mm. The plate is clamped with at all edges. Numerical results which are obtained by finite element simulation for the first five natural frequencies are presented in Table 3. The results in Table 3 indicate good agreement with the present method.

Table 3. Natural frequency of clamped (all edges)  $[0/\pm 45/90]_s$  laminated composite rectangular plate.

Mesh size	Mode number		
	1	2	3
4x4	86.535	134.82	218.55
10x10	84.642	132.86	206.72
15x15	84.86	133.42	205.95
20x20	84.936	133.62	206.07
25x25	84.992	133.78	206.23
30x30	85.026	133.87	206.34
35x35	85.041	133.91	206.4
40x40	85.051	133.93	206.44
100x100	85.085	134.02	206.58
Literature[28]	85.1	134	207.4

### 3.2. Finite element model details

A total 164 analyses are performed in this design study, using a finite element model of the plate. The model was developed using shell 99 element in ANSYS 10.0. The global x-coordinate is taken along the length of the plate; the global y-coordinate is taken along the width of the plate while the global z-direction is taken out the plate surface. The plate is analysed for natural frequencies under clamped boundary condition. In the present study, the *D-optimal* design has been implemented to select a feasible set of laminates from among all feasible laminates. Let us consider the case of a 16-ply laminate. The total number of entire feasible laminates is  $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 6561$  because we consider only a symmetric laminate and we adopt 3 levels ( $0^\circ$ ,  $45^\circ$  and  $90^\circ$ ) for each ply. The ‘odd’ occurrences i.e., first (outermost), third, fifth, etc. of  $45^\circ$  plies corresponds to  $45^\circ$  plies, whereas ‘even’ occurrences corresponds to  $-45^\circ$ . For example, a laminate of  $[0/45/45/90/45/90/90]$  is coded as  $[0/45/-45/90/45/90/90]$ . There is one unbalanced  $45^\circ$  ply when the number of occurrences of  $45^\circ$  plies is odd. This is repaired by replacing the  $45^\circ$  -ply with a  $90^\circ$ -ply or a  $0^\circ$ -ply. The  $45^\circ$ -ply

position replaced by a  $90^\circ$ -ply or a  $0^\circ$ -ply is the innermost  $45^\circ$ -ply that can be replaced without violating the four contiguous ply rule: the same fibre angle plies must not stacked more than four plies [5]. We can select feasible laminates from the set of feasible laminates using *D-optimal*. In this study, the total 164 feasible laminates were selected for modeling (134 laminates) and validation (30 laminates) of the RSM model. Because, forty five unknown coefficients exist in the quadratic polynomial function of eight variables (eight layers). Hence the number of experiments required for regression is approximately more twice the number of coefficients. The selected *D-optimal* set of laminates design was performed using Design Expert 8.0.2 software and the results are shown in Fig 2. The material properties used throughout this study are shown in Table 1. The natural frequencies are presented here in non-dimensional form using the Eq.(4):

$$\bar{\omega} = \omega \times \frac{b^2}{h} \sqrt{\frac{\rho}{E_2}} \quad (4)$$

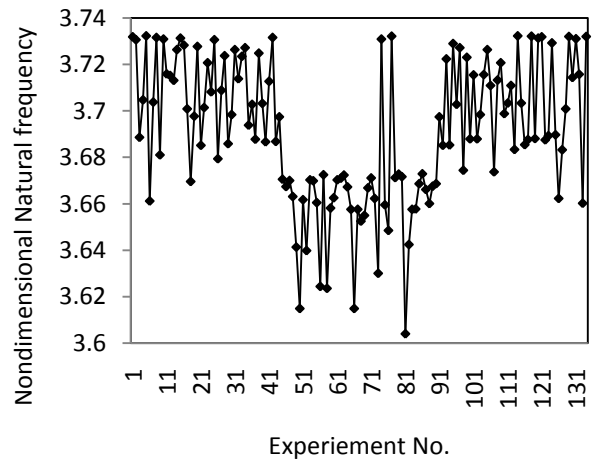


Fig 2. Experimental results for natural frequency

## 4. Results and discussion

### 4.1. Response surface analysis

The second order response representing the natural frequency can be expressed as a function of ply orientations. The relationship between the natural frequency and ply orientation has been expressed as follows

$$\begin{aligned} \text{Natural frequency} = & \beta_0 + \beta_1(X_1) + \beta_2(X_2) + \beta_3(X_3) + \beta_4(X_4) + \\ & \beta_5(X_5) + \beta_6(X_6) + \beta_7(X_7) + \beta_8(X_8) + \beta_9 X_1^2 + \beta_{10} X_2^2 + \beta_{11} X_3^2 + \\ & \beta_{12} X_4^2 + \beta_{13} X_5^2 + \beta_{14} X_6^2 + \beta_{15} X_7^2 + \beta_{16} X_8^2 + \beta_{17} \times X_1 \times X_2 + \\ & \beta_{18} \times X_1 \times X_3 + \beta_{19} \times X_1 \times X_4 + \beta_{20} \times X_1 \times X_5 + \beta_{21} \times X_1 \times X_6 + \beta_{22} \times \\ & X_1 \times X_7 + \beta_{23} \times X_1 \times X_8 + \beta_{24} \times X_2 \times X_3 + \beta_{25} \times X_2 \times X_4 + \beta_{26} \times X_2 \times \\ & X_5 + \beta_{27} \times X_2 \times X_6 + \beta_{28} \times X_2 \times X_7 + \beta_{29} \times X_2 \times X_8 + \beta_{30} \times X_3 \times X_4 + \end{aligned}$$

$$\beta_{31} \times X_3 \times X_5 + \beta_{32} \times X_3 \times X_6 + \beta_{33} \times X_3 \times X_7 + \beta_{34} \times X_3 \times X_8 + \beta_{35} \times X_4 \times X_5 + \beta_{36} \times X_4 \times X_6 + \beta_{37} \times X_4 \times X_7 + \beta_{38} \times X_4 \times X_8 + \beta_{39} \times X_5 \times X_6 + \beta_{40} \times X_5 \times X_7 + \beta_{41} \times X_5 \times X_8 + \beta_{42} \times X_6 \times X_7 + \beta_{43} \times X_6 \times X_7 + \beta_{44} \times X_7 \times X_8 \quad (5)$$

The second order response surface to estimate the natural frequency is created from finite element results for 134 laminates. A quadratic polynomial is employed for the response surface function in un-coded units as

$$\begin{aligned} \text{Natural frequency} = & 3.72094 - 9.58822E-04 \times x(1) + 0.000666007 \times x(2) + 0.000724131 \times x(3) + \\ & 0.000749135 \times x(4) + 0.000639712 \times x(5) + 0.000549168 \times x(6) + 0.000643228 \times x(7) + 0.000413358 \times x(8) + 1.15220E-05 \times x(1) \times x(1) - 5.82438E-06 \times x(2) \times x(2) - 7.34020E-06 \times x(3) \times x(3) - 7.85762E-06 \times x(4) \times x(4) - 6.75145E-06 \times x(5) \times x(5) - 6.82117E-06 \times x(6) \times x(6) - 6.68961E-06 \times x(7) \times x(7) - 3.54104E-06 \times x(8) \times x(8) - 1.31631E-06 \times x(1) \times x(2) + 2.81329E-07 \times x(1) \times x(3) - 1.02128E-07 \times x(1) \times x(4) - 3.66684E-07 \times x(1) \times x(5) + 2.94116E-07 \times x(1) \times x(6) + 3.37326E-07 \times x(1) \times x(7) - 1.05851E-06 \times x(1) \times x(8) - 1.33938E-07 \times x(2) \times x(3) - 4.77002E-07 \times x(2) \times x(4) - 2.58701E-07 \times x(2) \times x(5) - 1.77331E-07 \times x(2) \times x(6) + 1.71183E-07 \times x(2) \times x(7) - 4.51168E-07 \times x(2) \times x(8) - 1.70351E-07 \times x(3) \times x(4) - 2.00859E-09 \times x(3) \times x(5) - 1.14736E-07 \times x(3) \times x(6) - 4.32180E-07 \times x(3) \times x(7) - 2.75977E-07 \times x(3) \times x(8) - 2.69930E-07 \times x(4) \times x(5) + 3.99462E-07 \times x(4) \times x(6) + 2.95054E-07 \times x(4) \times x(7) - 4.00530E-07 \times x(4) \times x(8) + 5.37393E-07 \times x(5) \times x(6) - 5.04426E-07 \times x(5) \times x(7) + 1.53366E-07 \times x(5) \times x(8) - 2.11604E-07 \times x(6) \times x(7) + 5.40658E-07 \times x(6) \times x(8) - 8.21063E-07 \times x(7) \times x(8) \end{aligned} \quad (6)$$

The multiple regression coefficient of the second order model was found to be 0.913 and the adjusted coefficient of multiple regression coefficient  $R\text{-Sq (adj)} = 0.8702$ . The high  $R\text{-Sq (adj)}$  represents that the response surface has good fit to the natural frequency. Results of ANOVA for the response function natural frequency are presented in the Table 4. This analysis is carried out for a level of significance of 5% i.e., for a level of confidence of 95%.

From the analysis of Table 4, it is apparent that, the F calculated value is greater than the F table value ( $F_{0.05, 44, 89} = 1.5675$ ). Hence the second order response function developed is adequate. Fig 3 shows the comparison of experimental values with RSM predicted results.

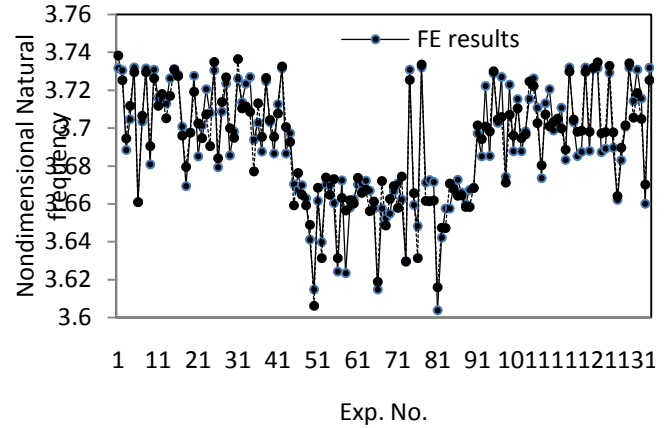


Fig3. Natural frequency vs. Experiment number for train data

#### 4.1. Validation of experimental results

To predict and verify the improvement in the natural frequency of laminated composite plate with respect to the chosen ply orientation setting, verification test is used. The validity of the response surface model was checked by the test data presented in Table 5 and is shown in Fig 4. From the analysis of Fig 4, it can be observed that the predicted values are very close to the experimental results.

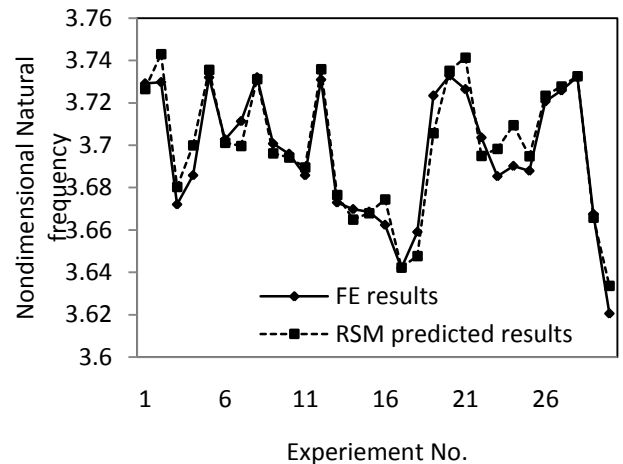


Figure 4. Natural frequency vs. Experiment number for validation data

#### 4.2. Regression Analysis

To have a more precise investigation into the model, a regression analysis of RSM predicted and experimentally measured values were performed and are shown in Fig 5-6. The adequacy of the developed model is verified by using the coefficient of determination ( $R^2$ ).  $0 \leq R^2 \leq 1$ . The  $R^2$  is the variability in the data accounted for by the model in percentage [14]. The regression coefficients are calculated to

estimate the correlation between the RSM predicted values and the experimentally measured values resulted from finite element tests. The regression coefficients are calculated by using Eq. (7).

$$R^2 = 1 - \left[ \frac{\sum_j (t_j - o_j)^2}{\sum_j (o_j)^2} \right] \quad (7)$$

Where  $t_j$ = targets or experimental values or measured values  
 $o_j$ = outputs or RSM predicted values.

There is a high correlation between the RSM predicted values and the experimentally measured values resulted from finite element analyses. The correlation coefficients for training and validation data are 0.913 and 0.92 respectively, which shows there is a strong correlation in modeling natural frequencies.

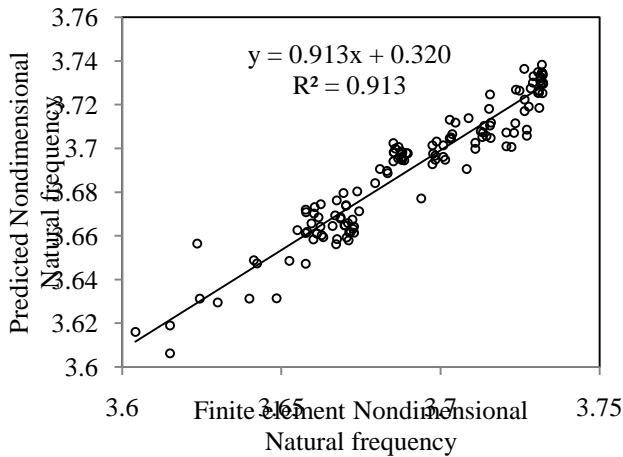


Fig 5. Regression analysis between predicted and Measured natural frequency for the training data set

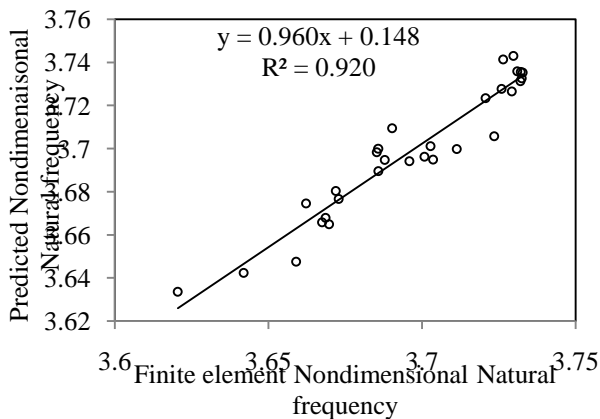


Fig 6. Regression analysis between predicted and Measured natural frequency for validation data set

### 4.3. Diagnostic checking of the developed model

The diagnostic checking of the developed model has been performed using residual analysis. The regression model was used to determine the residuals of each individual experimental run. The difference between the observed values and predicted or fitted values is called residuals. The normal probabilities of residuals are shown in Fig.7. The normal probability plot is used to verify the normality assumption. As shown in Fig 7, the data are spread roughly along the straight line. Hence it can be concluded that the data are normally distributed [19]. Fig.8 shows the residuals against the observation order. Fig.8 is used to show the correlation between the residuals. From the Fig.8 it is asserted that a tendency to have runs of positive and negative residuals indicates the existence of a certain correlation. Also the plot shows that the residuals are distributed evenly in both negative and positive along the run. Hence, the data can be said to be independent. Fig.9 indicates the residuals versus fitted values, which shows only the maximum variation of -0.04 to 0.02 in nondimensional natural frequency between observed and fitted values. This plot does not reveal any obvious pattern and hence the fitted model is adequate.

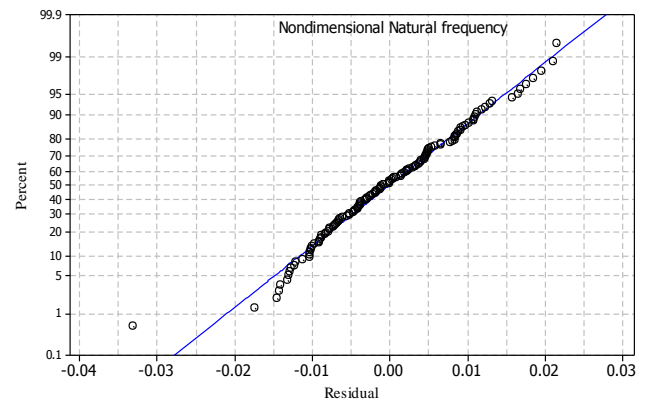


Fig 7. Normal probability plot of residuals

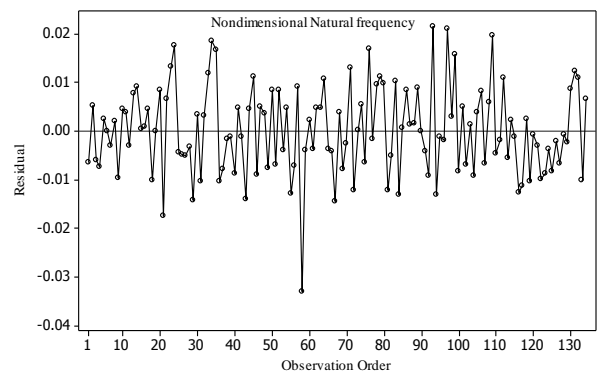


Fig 8. Residuals against the observed order of data

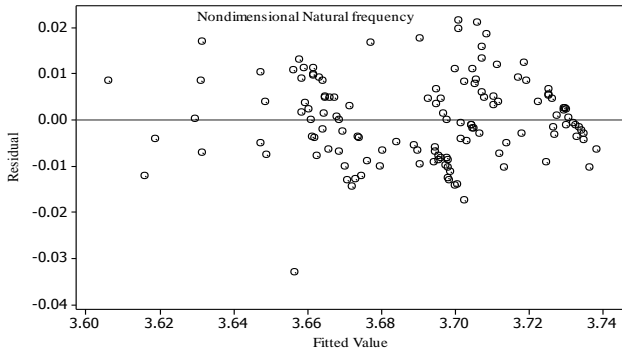


Fig 9. Residuals against the fitted values

**4.4. Parametric Analysis**

The linear layered structural shell element is used to predict the natural frequency of sixteen ply laminated composite plate under clamped boundary conditions. To study the effect of aspect ratios modulus ratios and side to thickness ratios on the natural frequency, the optimal stacking sequence is selected from the finite element results is [0/90/0/90/0/90/0/0] s. The results obtained for natural frequency from the finite element analysis are plotted in non dimensional quantities as a function of aspect ratios ( $b/a$ ), modulus ratios ( $E_1/E_2$ ) and side-to-thickness ratios ( $b/h$ ). In the analysis, the width of the plate and young modulus in the x-direction is changed. Figures 10 to 15 show the effect of the transverse shear deformation and bending extensional coupling and material orthotropy on the natural frequency of a sixteen ply symmetric laminated composite plate under clamped boundary condition. With the increase of the degree of orthotropy, increases the natural frequency. This is due to the fact that the Young's modulus of the material increases. From Fig.10 to Fig.15, it is observed that as the aspect ratio increases, the natural frequency increases. Larger the side to thickness ratios lesser the influence of natural frequency.

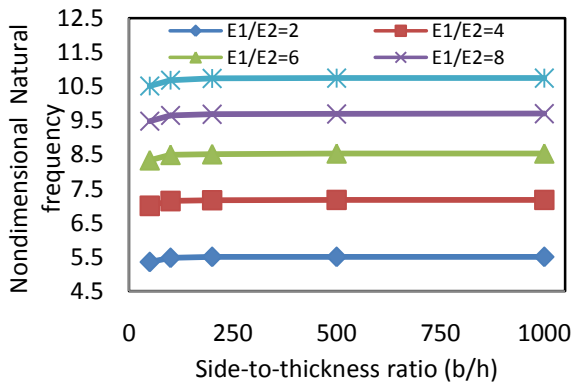


Fig.10. Nondimensional Frequency versus plate side to thickness ratio (Aspect ratio:  $b/a=2$ ) for different modulus ratios ( $E_1/E_2$ )

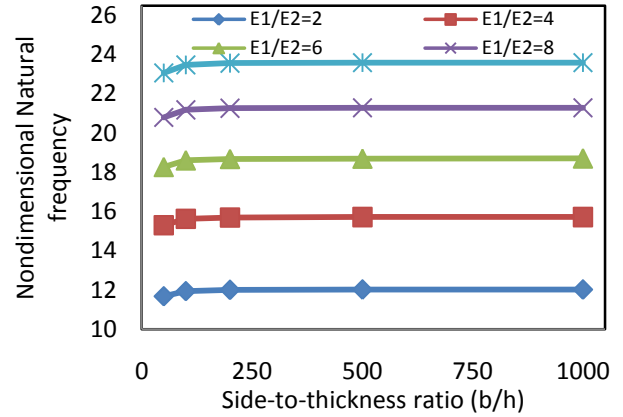


Fig.11. Nondimensional Frequency versus plate side to thickness ratio (Aspect ratio:  $b/a=3$ ) for different modulus ratios ( $E_1/E_2$ )

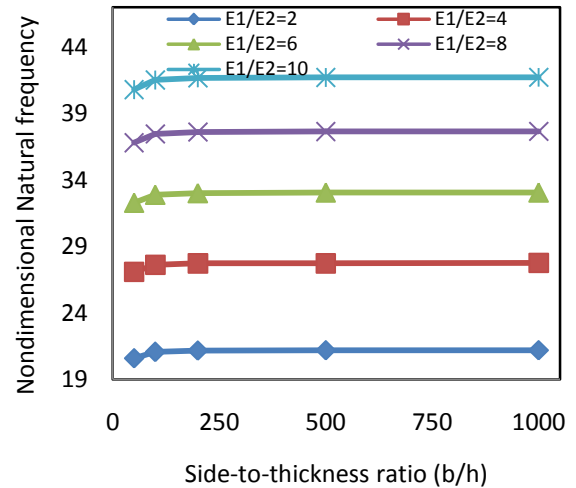


Fig.12. Nondimensional Frequency versus plate side to thickness ratio (Aspect ratio:  $b/a=4$ ) for different modulus ratios ( $E_1/E_2$ )

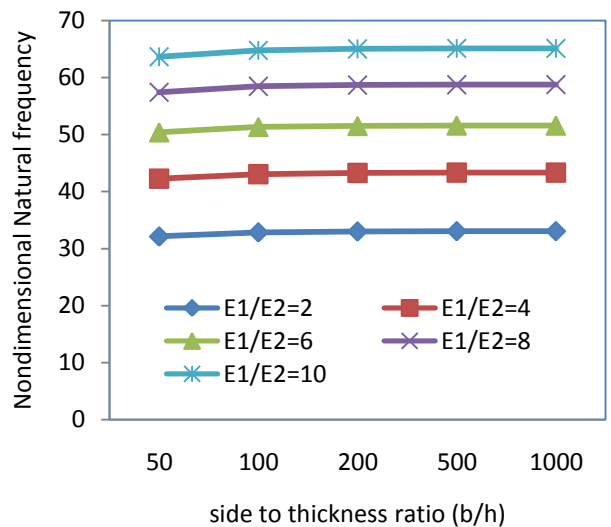


Fig.13. Nondimensional Frequency versus plate side to thickness ratio (Aspect ratio:  $b/a=5$ ) for different modulus ratios ( $E_1/E_2$ )

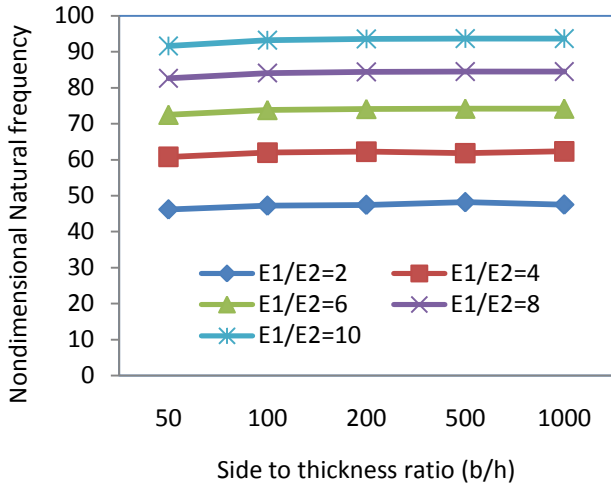


Fig. 14. Nondimensional Frequency versus plate side to thickness ratio (Aspect ratio:  $b/a=6$ ) for different modulus ratios ( $E_1/E_2$ )

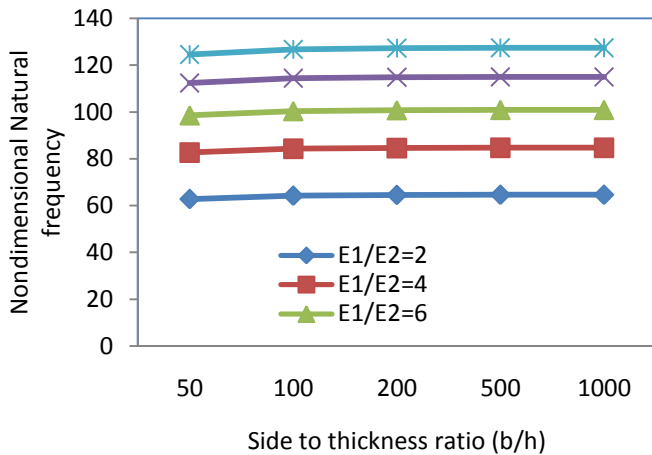


Fig. 15. Nondimensional Frequency versus plate side to thickness ratio (Aspect ratio:  $b/a=7$ ) for different modulus ratios ( $E_1/E_2$ )

## 5. Conclusions

This study presented a new  $D$ -optimal set of laminates and response surface methodology for creation of a response surface to predict the natural frequency of laminated composite plates. The  $D$ -optimal set of laminates and response surface methodology can be applied to any structural analysis to predict the any output responses. The proposed  $D$ -optimal set of laminates is not limited to 16-ply laminates. It is applicable to laminates of any number of plies by changing the ply thickness. The effectiveness of the method is shown with the natural frequency prediction of laminated composite plates.

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Table 4: ANOVA table for response function for natural frequency

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	44	0.113631	0.113631	0.002583	21.26	0.000
Residual Error	89	0.010812	0.010812	0.000121		
Total	133	0.124442				

Table 5. Test data and predicted results

S.No.	First ply angle	Second ply angle	Third ply angle	fourth ply angle	fifth ply angle	sixth ply angle	Seventh ply angle	Eighth ply angle	Nondimensional Natural frequency	Predicted Nondimensional Natural frequency
1.	0	0	90	0	0	0	90	90	3.729146	3.726491
2.	0	90	90	0	90	0	0	90	3.729677	3.742979
3.	0	45	90	-45	45	90	-45	90	3.671947	3.680327
4.	0	45	-45	90	90	0	0	90	3.685729	3.699937
5.	0	90	0	90	90	0	90	0	3.732056	3.735613
6.	0	90	45	-45	90	90	90	90	3.702762	3.701105
7.	0	0	90	45	90	-45	90	90	3.711249	3.699729
8.	0	90	90	90	0	90	90	90	3.731982	3.731288
9.	0	0	45	0	-45	0	0	0	3.700776	3.696204
10.	0	90	45	0	-45	45	0	-45	3.695861	3.694164
11.	0	45	0	-45	90	90	0	0	3.685813	3.689579
12.	90	0	90	0	90	0	0	0	3.730948	3.735867
13.	45	-45	0	0	90	90	0	90	3.672857	3.67659

14.	45	90	-45	90	90	0	90	0	3.669757	3.664905
15.	45	-45	90	90	90	0	90	90	3.668557	3.667935
16.	45	0	90	90	0	-45	90	0	3.66228	3.674502
17.	45	-45	0	45	0	-45	90	90	3.641979	3.642319
18.	45	0	-45	90	90	45	0	-45	3.659012	3.647619
19.	90	0	90	0	45	-45	45	-45	3.723431	3.705749
20.	90	0	90	90	0	90	0	90	3.73273	3.735242
21.	90	90	90	90	0	0	90	0	3.726402	3.7414
22.	90	0	45	-45	90	90	90	90	3.703542	3.69491
23.	90	45	90	-45	90	90	0	0	3.685267	3.698305
24.	90	45	0	0	0	-45	90	0	3.69021	3.709446
25.	90	45	-45	90	90	90	0	0	3.68787	3.694786
26.	0	0	0	0	0	90	90	90	3.720637	3.72335
27.	0	0	0	0	90	90	90	90	3.72581	3.727746
28.	0	90	0	90	90	90	0	0	3.732373	3.732543
29.	45	-45	0	0	0	0	90	90	3.667503	3.66581
30.	45	-45	45	-45	90	90	90	90	3.62051	3.633605