International Journal of Scientific Engineering and Technology (ISSN : 2277-1581)
Volume No.3 Issue No.3, pp : 211 – 215 1 March 2014

Some Features of α-T_2 Space in Intuitionistic Fuzzy Topology

Rafiqul Islam

Department of Mathematics, Pabna University of Science and Technology, Pabna-6600, Bangladesh

Email: rafiqul.pust.12@gmail.com

Abstract : The basic concepts of the theory of intuitionistic fuzzy topological spaces have been defined by D. Coker and co-workers. In this paper, we define new notions of Hausdorffness in the intuitionistic fuzzy sense, and obtain some new properties "good extension property" is one of them, in particular on convergence.

MSC (2000): 54A40, 03E72.

Key words: Fuzzy set, Intuitionistic fuzzy topology, Hausdorffness, IFS.

Introduction

The introduction of "intuitionistic fuzzy sets" is due to K.T Atanassov [1], and this theory has been developed by many authors [2-4]. In particular D. Coker has defined the intuitionistic fuzzy topological spaces, and several authors have studied this category [5-14]. Nevertheless, separation in intuitionistic fuzzy topological spaces is not studied. Only there exists a definition due to D. Coker.

Definition: Let X be a non-empty set and I=[0,1]. A fuzzy set in X is a function $u: X \to I$ which assign to each element $x \in X$, a degree of membership, $u(x) \in I$.

Example: Let $X=\{a,b,c\}$ and $I=[0,1]$.If $u(a)=0.2, u(b)=0.4, u(c)=0.5$ then {(a,0.2),(b,0.4),(c,0.5)} is a fuzzy set in X.

Definition: Let $I=[0,1]$. X be a non-empty set and I^X be the collection of all mappings from X into I, i. e. the class of all fuzzy sets in X. A fuzzy topology on X is defined as a family t of members of I^X, satisfying the following conditions: $(i) 1,0 \in t$ (ii) if $u_i \in t$ for each $i \in \Delta$, then $\cup_{i\in\Delta} u_i \in t$ (iii) if $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$. Then the pair (X,t) is called a fuzzy topological space (FTS) and the members of t are called t-open (or simply open) fuzzy sets. A fuzzy set v is called a t-closed (or simply closed) fuzzy set if $1-v \in t$.

Example:

Let $X=\{a,b,c,d\}$, $t=\{\underline{0},\underline{1},u,v\}$, where

$\underline{1}=\{(a,1),(b,1),(c,1),(d,1)\}$

$\underline{0}=\{(a,0),(b,0),(c,0),(d,0)\}$

$u=\{(a,0.2),(b,0.5),(c,0.7),(d,0.9)\}$

$v=\{(a,0.3),(b,0.5),(c,0.8),(d,0.95)\}$ Then (X,t) is a fuzzy topological space.

Definition: (Atanassov [4]). Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS) A is an object having the form $A=\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

Definition: Let X be a nonempty set and τ be a family of intuitionistic fuzzy sets in X. Then τ is called an intuitionistic fuzzy topology on X if it satisfy the following conditions: $(i) \underline{0}, \underline{1} \in \tau$

$(ii) G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

$(iii) \cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$. In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X.

Definition: An IFTS (X,τ) is called Hausdorff iff $x_1, x_2 \in X$ and $x_1 \ne x_2$ imply that there exist $G_1 = \langle x, \mu_{G_1}, \gamma_{G_1}\rangle, G_2 = \langle x, \mu_{G_2}, \gamma_{G_2}\rangle \in \tau$ with $\mu_{G_1}(x_1)=1, \gamma_{G_1}(x_1)=0$ $\mu_{G_2}(x_2)=1, \gamma_{G_2}(x_2)=0$ and $G_1 \cap G_2 = \underline{0}$.

Definition: An IFTS (X,τ) is called (a) $T_2(i)$ if for all $x_1, x_2 \in X$, $x_1 \ne x_2$ imply that there exist open sets $G_1 = \langle x, \mu_{G_1}, \gamma_{G_1}\rangle, G_2 = \langle x, \mu_{G_2}, \gamma_{G_2}\rangle \in \tau$ such that $\mu_{G_1}(x_1)=1, \gamma_{G_1}(x_1)=0$ $\mu_{G_2}(x_2)=1, \gamma_{G_2}(x_2)=0$ and $G_1 \cap G_2 = \underline{0}$.

(b) $\alpha - T_2(ii)$ if for all $x_1, x_2 \in X$ with $x_1 \neq x_2$ imply that there exist open sets $G_1 = \langle x, \mu_{G_1}, \gamma_{G_1} \rangle, G_2 = \langle x, \mu_{G_2}, \gamma_{G_2} \rangle \in \tau$ such that $\mu_{G_1}(x_1) = 1, \gamma_{G_1}(x_1) = 0$ $\mu_{G_2}(x_2) = 1, \gamma_{G_2}(x_2) = 0$ and $G_1 \cap G_2 \leq \alpha$.

(c) $\alpha - T_2(iii)$ if for all $x_1, x_2 \in X$, $x_1 \neq x_2$

imply that there exists $G_1 = \langle x, \mu_{G_1}, \gamma_{G_1} \rangle, G_2 = \langle x, \mu_{G_2}, \gamma_{G_2} \rangle \in \tau$ such that $\mu_{G_1}(x_1) > \alpha, \mu_{G_2}(x_2) > \alpha$ and $G_1 \cap G_2 = \underline{0}$.

(d) $\alpha - T_2(iv)$ if for all $x_1, x_2 \in X$, $x_1 \neq x_2$

imply that there exist

$G_1 = \langle x, \mu_{G_1}, \gamma_{G_1} \rangle, G_2 = \langle x, \mu_{G_2}, \gamma_{G_2} \rangle \in \tau$ such that $\mu_{G_1}(x_1) > \alpha, \mu_{G_2}(x_2) > \alpha$ and $G_1 \cap G_2 \leq \alpha$.

Theorem: If (X, T) be fuzzy topological space and (X, τ) be corresponding intuitionistic fuzzy topological space(IFTS) then (X,T) is $\alpha - T_2(j) \Rightarrow (X, \tau)$ is $\alpha - T_2(j)$,for $J = i, ii, iii, iv.$

But the converse is not true.

Proof: First suppose that (X,T) is a fuzzy $\alpha - T_2(ii)$ space. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since (X,T) is fuzzy $\alpha - T_2(ii)$ space,for some $\alpha \in I_1$, $\forall x_1, x_2 \in X$ with $x_1 \neq x_2$,$\exists u, v \in T$ such that $u(x_1) = 1 = v(x_2)$ and $u \cap v \leq \alpha$ .This implies that if for all $x_1, x_2 \in X$, $x_1 \neq x_2$ imply that there exist open sets $G_1 = \langle x, \mu_{G_1}, \gamma_{G_1} \rangle, G_2 = \langle x, \mu_{G_2}, \gamma_{G_2} \rangle \in \tau$ such that $\mu_{G_1}(x_1) = 1, \gamma_{G_1}(x_1) = 0$ $\mu_{G_2}(x_2) = 1, \gamma_{G_2}(x_2) = 0$ and $G_1 \cap G_2 \leq \alpha$ .Hence we have (X, τ) is $\alpha - T_2(ii)$ space. Similarly, one can see that (X,T) is $\alpha - T_2(i) \Rightarrow (X, \tau)$ is $\alpha - T_2(i)$.(X,T) is $\alpha - T_2(iii) \Rightarrow$ (X, τ) is $\alpha - T_2(iii)$.

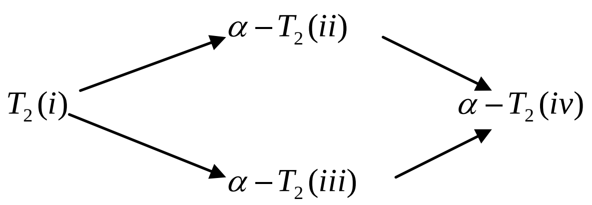
(X,T) is $\alpha - T_2(iv) \Rightarrow$ (X, τ) is $\alpha - T_2(iv)$.

Example: Let $X = \{x_1, x_2\}$ and (X,T) be the fuzzy topology on X generated by $\{u, v\} \cup \{constants\}$,where $u(x_1) = 1, u(x_2) = 0, v(x_1) = 0, v(x_2) = 1$ .Again let τ be the indiscreat topology on X.Then for every $\alpha \in I_1$, the IFTS (X, τ) is $\alpha - T_2(j)$. But the fuzzy topological space (X,T) is not $\alpha - T_2(j)$ for $J = i, ii, iii, iv.$

Remarks: Let (X,T) be the fuzzy topological space and (X, τ) be its corresponding IFTS. Then (X, τ) is $\alpha - T_2(j)$ does not imply (X,T) is $\alpha - T_2(j)$ for $J = i, ii, iii, iv.$ For this consider the following example.

Example: Let $X = \{x, y\}$ and T be the fuzzy topology on X generated by $\{u\} \cup \{constants\}$,where $u(x) = 1, u(y) = 0$. Again let τ be the intuitionistic fuzzy topology on X generated by $\{G_1\} \cup \{constants\}$,where $\mu_{G_1}(x) = 1, \gamma_{G_1}(x) = 0$ .Then for every $\alpha \in I_1$, the IFTS (X, τ) is $\alpha - T_2(j)$. But the fuzzy topological space (X,T) is not $\alpha - T_2(j)$ for $J = i, ii, iii, iv.$

Theorem: Let (X, τ) be an IFTS. Then we have the following implication:



Proof: Let (X, τ) be $T_2(i)$ .We prove that (X, τ) is $\alpha - T_2(ii)$. Let $x_1, x_2 \in X$, $x_1 \neq x_2$. Since (X, τ) is $T_2(i)$, there exist open sets $G_1 = \langle x, \mu_{G_1}, \gamma_{G_1} \rangle, G_2 = \langle x, \mu_{G_2}, \gamma_{G_2} \rangle \in \tau$ such that $\mu_{G_1}(x_1) = 1, \gamma_{G_1}(x_1) = 0$, $\mu_{G_2}(x_2) = 1, \gamma_{G_2}(x_2) = 0$ and $G_1 \cap G_2 = \underline{0}$. We see that that $\mu_{G_1}(x_1) > \alpha, \mu_{G_2}(x_2) > \alpha$, and $G_1 \cap G_2 \leq \alpha$ for every $\alpha \in I_1$ .Hence it is clear that (X, τ) is $\alpha - T_2(ii)$ and also $\alpha - T_2(iii)$.

Further one can easily verify that

$$\alpha - T_2(ii) \Rightarrow \alpha - T_2(iv)$$

$$\alpha - T_2(iii) \Rightarrow \alpha - T_2(iv)$$

$T_2(i) \Rightarrow \alpha - T_2(iii)$

Now we give some examples to show that none of the reverse implications are true in general.

Example(a): Let $X = \{x_1, x_2\}$ and $G_1, G_2 \in (I \times I)^X$ where G_1, G_2 are defined by $\mu_{G_1}(x_1) = 0.7, \gamma_{G_1}(x_1) = 0$ and $\mu_{G_2}(x_2) = 0, \gamma_{G_2}(x_2) = 0.8$. Consider the intuitionistic fuzzy topology τ on X generated by $\{G_1, G_2\} \cup \{constants\}$. For $\alpha = 0.4$, it is clear that (X,τ) is $\alpha - T_2(iii)$ but (X,τ) is neither $\alpha - T_2(ii)$ nor $T_2(i)$.

Example *(b):* Let $X = \{x_1, x_2\}$ and $G_1, G_2 \in (I \times I)^X$ where G_1, G_2 are defined by $\mu_{G_1}(x_1) = 1, \gamma_{G_1}(x_1) = 0$ and $\mu_{G_2}(x_2) = 0, \gamma_{G_2}(x_2) = 1$. Consider the intuitionistic fuzzy topology τ on X generated by $\{G_1, G_2\} \cup \{constants\}$. For $\alpha = 0.5$, it is clear that (X,τ) is $\alpha - T_2(ii)$ but (X,τ) is neither $\alpha - T_2(iii)$ nor $T_2(i)$.

Example(c)*:* Let $X = \{x_1, x_2\}$ and $G_1, G_2 \in (I \times I)^X$ where G_1, G_2 are defined by $\mu_{G_1}(x_1) = 0.8, \gamma_{G_1}(x_1) = 0$ and $\mu_{G_2}(x_2) = 0.5, \gamma_{G_2}(x_2) = 0.3$. Consider the intuitionistic fuzzy topology τ on X generated by $\{G_1, G_2\} \cup \{constants\}$. For $\alpha = 0.4$, it is clear that (X,τ) is $\alpha - T_2(iv)$ but (X,τ) is neither $\alpha - T_2(ii)$ nor $\alpha - T_2(iii)$.

Theorem: If (X,τ) is IFTS and $0 \le \alpha \le \beta < 1$ then

(a) $\alpha - T_2(ii) \Rightarrow \beta - T_2(ii)$

(b) $\beta - T_2(iii) \Rightarrow \alpha - T_2(iii)$

(c) $0 - T_2(iii) \Rightarrow 0 - T_2(iv)$

Proof: Let (X,τ) be $\alpha - T_2(ii)$. We prove that (X,τ) is $\beta - T_2(ii)$. Since (X,τ) is $\alpha - T_2(ii)$, if for all $x_1, x_2 \in X$, $x_1 \ne x_2$ imply that there exist open sets $G_1 = \langle x, \mu_{G_1}, \gamma_{G_1} \rangle, G_2 = \langle x, \mu_{G_2}, \gamma_{G_2} \rangle \in \tau$ such that $\mu_{G_1}(x_1) = 1, \gamma_{G_1}(x_1) = 0$, $\mu_{G_2}(x_2) = 1, \gamma_{G_2}(x_2) = 0$ and $G_1 \cap G_2 \le \alpha$. This implies that $\mu_{G_1}(x_1) = 1, \gamma_{G_1}(x_1) = 0$, $\mu_{G_2}(x_2) = 1, \gamma_{G_2}(x_2) = 0$ and $G_1 \cap G_2 \le \beta$ as $0 \le \alpha \le \beta < 1$. Hence it is clear that (X,τ) is $\beta - T_2(ii)$.

Example: Let $X = \{x_1, x_2\}$ and $G_1, G_2 \in (I \times I)^X$ where G_1, G_2 are defined by $\mu_{G_1}(x_1) = 1, \gamma_{G_1}(x_1) = 0$ and $\mu_{G_2}(x_2) = 1, \gamma_{G_2}(x_2) = 0$. Consider the intuitionistic fuzzy topology τ on X generated by $\{G_1, G_2\} \cup \{constants\}$. For $\alpha = 0.5, \beta = 0.8$, it is clear that (X,τ) is $\beta - T_2(ii)$ but (X,τ) is not $\alpha - T_2(ii)$. Further one can easily verify that $\beta - T_2(iii) \Rightarrow \alpha - T_2(iii)$ and $0 - T_2(iii) \Rightarrow 0 - T_2(iv)$ are true.

This completes the proof.

'Good extension' property

Now we discuss about the "good extension" property of $T_2(j)$ for j = i , ii ,iii , iv.

Definition: Let f be a real valued function on a topological space. If $\{x : f(x) > \alpha\}$ is open for every real α, then f is called lower semi continuous function.

Definition: Let X be a non-empty set and t be a topology on X. Let $\tau = \omega(t)$ be the set of all lower semi continuous function (lsc) from (X, t) to $(I \times I)$ (with usual topology). Thus

$\omega(t) = \{G \in (I \times I)^X : [\mu_G^{-1}(\alpha, 1], \gamma_G^{-1}[0, \alpha)]\}$ where $\mu_G : X \to I, \gamma_G : X \to I$ for each $\alpha \in I_1$. It can be shown that $\omega(t)$ is a intuitionistic fuzzy topology on X.

Let P be the property of a topological space (X , t) and FP be its intuitionistic fuzzy topological analogue. Then FP is called a "good extension" of P " iff the statement (X, t) has P iff $(X, \omega(t))$ has FP" holds good for every topological space (X, t).

Theorem: Let (X, t) be a IFTS. Consider the following statements:

(1). (X, t) be $T_2(i)$ space.

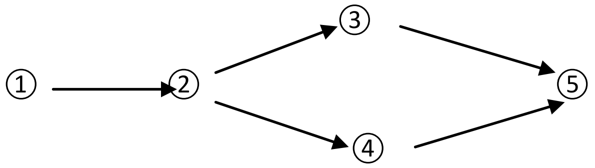
(2). $(X, \omega(t))$ be $T_2(i)$ space.

(3). $(X,\omega(t))$ be $\alpha-T_2(ii)$ space.

(4). $(X,\omega(t))$ be $\alpha-T_2(iii)$ space.

(5). $(X,\omega(t))$ be $\alpha-T_2(iv)$ space.

Then the following implications are true



Proof: Let the intuitionistic fuzzy topological space (X, t) be $T_2(i)$. Suppose $x_1,x_2\in X$ with $x_1\neq x_2$. Since (X, t) is $T_2(i)$, there exist $G_1=\langle x,\mu_{G_1},\gamma_{G_1}\rangle, G_2=\langle x,\mu_{G_2},\gamma_{G_2}\rangle\in t$ such that $\mu_{G_1}(x_1)=1,\gamma_{G_1}(x_1)=0$, $\mu_{G_2}(x_2)=1,\gamma_{G_2}(x_2)=0$ and $G_1\cap G_2=\underline{0}$. But from the definition of the lower semi continuous function, there exist $1_{G_1},1_{G_2}\in\omega(t)$ such that $1_{\mu_{G_1}(x_1)}=1, 1_{\gamma_{G_1}(x_1)}=0, 1_{\mu_{G_2}(x_2)}=1, 1_{\gamma_{G_2}(x_2)}=0$,and $1_{G_1\cap G_2}=0$ i, e $1_{G_1}\cap 1_{G_2}=0$. Hence it is clear that the IFTS $(X,\omega(t))$ is $T_2(i)$ space.

Further it can be easily to show that $(2)\to(4)$ $(2)\to(3)$ $(3)\to(5)$ and $(4)\to(5)$ .Hence proved.

Theorem: Let (X,t) be a IFTS and

$$I_\alpha(t)=\{\langle \mu_{G_1}{}^{-1}(\alpha,1],\gamma_{G_1}{}^{-1}[0,\alpha)\rangle, \langle \mu_{G_2}{}^{-1}(\alpha,1],\gamma_{G_2}{}^{-1}[0,\alpha)\rangle : G_1,G_2\in t\}$$

then

(a) (X, t) is $\alpha-T_2(ii)\Rightarrow(X,I_\alpha(t))$ is $T_2(i)$

(b) (X, t) is $\alpha-T_2(iii)\Rightarrow(X,I_\alpha(t))$ is $T_2(i)$

(c) (X, t) is $\alpha-T_2(iv)\Leftrightarrow(X,I_\alpha(t))$ is $T_2(i)$

The reverse implications in (a) and (b) are not true in general.

Proof: Let the intuitionistic fuzzy topological space (IFTS in short) (X, t) be a $\alpha-T_2(ii)$.We shall prove that the topological space $(X,I_\alpha(t))$ is $T_2(i)$. Since (X, t) is $\alpha-T_2(ii)$, if for all $x_1,x_2\in X$, $x_1\neq x_2$ imply that there exist open sets $G_1=\langle x,\mu_{G_1},\gamma_{G_1}\rangle, G_2=\langle x,\mu_{G_2},\gamma_{G_2}\rangle\in t$ such that $\mu_{G_1}(x_1)=1,\gamma_{G_1}(x_1)=0$, $\mu_{G_2}(x_2)=1,\gamma_{G_2}(x_2)=0$ and $G_1\cap G_2\leq\alpha$ .But for every $\alpha\in I_1$,

$$\{\langle \mu_{G_1}{}^{-1}(\alpha,1],\gamma_{G_1}{}^{-1}[0,\alpha)\rangle, \langle \mu_{G_2}{}^{-1}(\alpha,1],\gamma_{G_2}{}^{-1}[0,\alpha)\rangle : G_1,G_2\in t\}\in I_\alpha(t)$$

and also $x_1\in\mu_{G_1}{}^{-1}(\alpha 1], x_2\in\mu_{G_2}{}^{-1}(\alpha,1]$ and $\mu_{G_1}{}^{-1}(\alpha,1]\cap\mu_{G_2}{}^{-1}(\alpha,1]=\phi$, as $G_1\cap G_2\leq\alpha$ .Hence it is clear that $(X,I_\alpha(t))$ is $T_2(i)$. Further , one can easily verify that

(X, t) is $\alpha-T_2(iii)\Rightarrow(X,I_\alpha(t))$ is $T_2(i)$ and (X, t) is $\alpha-T_2(iv)\Rightarrow(X,I_\alpha(t))$ is $T_2(i)$.

Conversely, suppose that $(X,I_\alpha(t))$ is $T_2(i)$ .Let $x_1,x_2\in X$, $x_1\neq x_2$.Since $(X,I_\alpha(t))$ is $T_2(i)$ there,exist

$$\{\langle \mu_{G_1}{}^{-1}(\alpha,1],\gamma_{G_1}{}^{-1}[0,\alpha)\rangle, \langle \mu_{G_2}{}^{-1}(\alpha,1],\gamma_{G_2}{}^{-1}[0,\alpha)\rangle : G_1,G_2\in t\}\in I_\alpha(t)$$

such that

$x_1\in\mu_{G_1}{}^{-1}(\alpha,1]$, $x_2\in\mu_{G_2}{}^{-1}(\alpha,1]$ and $\mu_{G_1}{}^{-1}(\alpha,1]\cap\mu_{G_2}{}^{-1}(\alpha,1]=\phi$. Again since

$$\{\langle \mu_{G_1}{}^{-1}(\alpha,1],\gamma_{G_1}{}^{-1}[0,\alpha)\rangle, \langle \mu_{G_2}{}^{-1}(\alpha,1],\gamma_{G_2}{}^{-1}[0,\alpha)\rangle : G_1,G_2\in t\}\in I_\alpha(t)$$

, so we get $G_1=\langle x,\mu_{G_1},\gamma_{G_1}\rangle, G_2=\langle x,\mu_{G_2},\gamma_{G_2}\rangle\in t$ such that $\mu_{G_1}(x_1)>\alpha,\mu_{G_2}(x_2)>\alpha$ and $\mu_{G_1}{}^{-1}(\alpha,1]\cap\mu_{G_2}{}^{-1}(\alpha,1]=\phi\Rightarrow(\mu_{G_1}\cap\mu_{G_2})^{-1}(\alpha,1]=\phi$ $i.e, G_1\cap G_2\leq\alpha.$ So we see that (X, t) is $\alpha-T_2(iv)$.

Now we have an example for non-implication.

Example: Let $X=\{x_1,x_2\}$ and $G_1,G_2\in(I\times I)^X$ where G_1,G_2 are defined by $\mu_{G_1}(x_1)=0.8,\gamma_{G_1}(x_1)=0.2$ and $\mu_{G_2}(x_2)=0.1,\gamma_{G_2}(x_2)=0.6$. Consider the intuitionistic fuzzy topology t on X generated by $\{G_1, G_2\} \cup \{constants\}$. For $\alpha = 0.4$, it is clear that (X, t) is neither $\alpha - T_2(ii)$ nor $\alpha - T_2(iii)$ .Now

$$I_\alpha(t) = \{\left\langle \mu_{G_1}{}^{-1}(\alpha,1], \gamma_{G_1}{}^{-1}[0,\alpha) \right\rangle, \left\langle \mu_{G_2}{}^{-1}(\alpha,1], \gamma_{G_2}{}^{-1}[0,\alpha) \right\rangle : G_1, G_2 \in t\}$$

Also, $x_1 \in \mu_{G_1}{}^{-1}(\alpha,1]$, $x_2 \in \mu_{G_2}{}^{-1}(\alpha,1]$ and $\mu_{G_1}{}^{-1}(\alpha,1] \cap \mu_{G_2}{}^{-1}(\alpha,1] = \phi$, as $G_1 \cap G_2 \leq \alpha$. Hence it is clear that $(X, I_\alpha(t))$ is $T_2(i)$.

This completes the proof.

References:

i. *K.T Atanassov: intuitionistic fuzzy sets, in VII ITKR's Session, Sofia (June 1983).*

ii. *K.T Atanassov: intuitionistic fuzzy sets, Fuzzy sets Syst. 20 (1986) , 87-96*

iii. *K.T Atanassov : Review and new results on intuitionistic fuzzy sets, preprint IM-MFAIS-1-88, Sofia (1988).*

iv. *K.T Atanassov: intuitionistic fuzzy sets. Theory and applications, Springer-Verlag (Heidelberg , New York , 1999)*

v. *D. Coker: An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets Syst. 88 (1997) , 81-89*

vi. *D. Coker: An introduction to fuzzy subspaces in intuitionistic fuzzy topological spaces , J. Fuzzy Math. 4 (1996), 749-764.*

vii. *D. Coker and M. Demirci: On intuitionistic fuzzy points, Notes IFS 1, 2 (1995), 79-84.*

viii. *D. Coker and A.H. Es: On fuzzy compactness in intuitionistic fuzzy topological spaces, J. Fuzzy Math. 3 (1995), 899-909.*