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A Numerical Simulator for Solving Numerical Integration

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Abstract : ***Numerical integration is a frequently-needed tool in modern Science and Engineering. Engineers and Scientists typically visualize integration as the process of determining the area under a curve. Besides this, because of its many more applications, it is often viewed as a discipline in and of itself. In this paper we develop a mathematical simulator for solving numerical integration problems. This simulator is incorporated with a combination of Trapezoidal rule, Simpson's*** $1/3$ ***rule, Simpson's*** $3/8$ ***rule and rule.***

Keywords : Trapezoidal, Weddle, Syracuse, language

1. Introduction

Integration generally means combining parts so that they form a whole. The foundation for the discovery of the integral was laid by Cavelieri, an Italian mathematician, in around 1635. Besides, numerical algorithms are almost as old as human civilization. In numerical analysis, numerical integration constitutes a broad family of algorithms for calculating the numerical value of a definite integral. Archimedes of Syracuse (287-212 BC) created much new mathematics, including the "method of exhaustion" for calculating lengths, areas, and volumes of geometric figures [1]. When used as a method to find approximations, it is in much the spirit of modern numerical integration; and it was an important precursor to the development of the calculus by Isaac Newton and Gottfried Leibnitz. Following Newton, many of the giants of mathematics of the 18^{th} and 19^{th} centuries made major contributions to the numerical solution of mathematical problems.

Given a set of data of points (x_0, y_0), (x_1, y_1), ..., (x_n, y_n) of a function $y = f(x)$ where $f(x)$ is not known explicitly, it is required to compute the value of the definite integral

$$I = \int_a^b y\ dx \qquad (1)$$

We derive a general formula for numerical integration using Newton's forward difference formula.

Let the interval $[a,b]$ be divided into n equal subintervals such that $a = x_0 < x_1 < ... < x_n = b$.

Clearly, $x_n = x_0 + nh$. Hence the integral becomes

$I = \int_{x_0}^{x_n} y\ dx$. Now approximating y by Newton's forward difference formula, we obtain [3]

$$I = \int_{x_0}^{x_n} [y_0 + p\,\Delta y_0 + \frac{p(p-1)}{1.2}\Delta^2 y_0 + \frac{p(p-1)(p-3)}{1.2.3}\Delta^3 y_0 + ...]\ dx$$

Since $x = x_0 + ph$ and so $dx = dp$ hence the above integral becomes

$$I = \int_0^n [y_0 + p\,\Delta y_0 + \frac{p(p-1)}{1.2}\Delta^2 y_0 + \frac{p(p-1)(p-3)}{1.2.3}\Delta^3 y_0 + \ldots]\ dp$$

This implies

$$\int_{x_0}^{x_n} y\ dx = nh[y_0 + (n/2)\,\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \ldots]dp \qquad (2)$$

From this general formula, we can obtain different integration formula by putting $n = 1, 2, 3, \ldots$ etc.

Programming language C is very flexible and powerful. It originally designed in the early 1970s [4]. It allows us to maximum control with minimum command. It is recognized worldwide and used in a multitude of applications especially in numerical analysis. Along with other numerous benefits, we have used programming language C in this paper.

The outline of this paper is as follows: Section 2 contains the brief description of the existing methods with methodology and simple examples. In Section 3, we develop a numerical simulator, using the programming language C, which gives us the solution of a problem simultaneously regarding four popular existing numerical integration methods namely Trapezoidal rule, Simpson's $1/3$ rule, Simpson's $3/8$ rule and Weddle's rule. Moreover, the simulator identifies the method that gives the best solution comparing with possible exact solution of the problem.

We devote Section 4 to an output of the program for a specific problem. Conclusions are given at the end at Section 5.

2. Existing Methods

We give a brief description of the existing methods of numerical integration like Trapezoidal rule, Simpson's $1/3$ rule, Simpson's $3/8$ rule and Weddle's rule in this section with their methodology and simple examples.

2.1 Trapezoidal Rule

Putting $n=1$ in (2) all differences higher than the first will become zero and therefore we have

$$\int_{x_0}^{x_1} y\ dx = h[y_0 + (1/2)\Delta y_0] = (h/2)[y_0 + y_1]$$

For the next interval $[x_1,\ x_2]$ and others we have the similar expression as $\int_{x_1}^{x_2} y\ dx = (h/2)[y_1 + y_2]$ and so on and finally for the last interval $[x_{n-1}, x_n]$, we have

$$\int_{x_{n-1}}^{x_n} y\ dx = (h/2)[y_{n-1} + y_n].$$

Combining all these expressions, we obtain the rule [2]

$$\int_{x_0}^{x_n} y\ dx = (h/2)[y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n]$$

This is known as Trapezoidal rule.

2.1.1 Example

We are estimating $\int_{-1}^{3} \frac{1}{\ln x + 3} dx$ using the Trapezoidal Rule with $n = 4$.

Here $h = (b-a)/n = (3+1)/4 = 1$, and

$$\int_{-1}^{3} \frac{1}{\ln x + 3} dx = (h/2)[y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n]$$

$$= [0.9102 + 2(0.721 + 0.621 + 0.558) + 0.514]/2 = 2.613$$

2.2 Simpson's 1/3 Rule

Putting $n = 2$ in (2) all differences higher than the second term will become zero and therefore we have

$$\int_{x_0}^{x_2} y\ dx = 2h[y_0 + \Delta y_0 + \Delta^2 y_0/6] = (h/3)[y_2 + 4y_1 + y_0]$$

Similarly, $\int_{x_2}^{x_4} y\ dx = (h/3)[y_4 + 4y_3 + y_2]$ and finally

$$\int_{x_{n-2}}^{x_n} y\ dx = (h/3)[y_n + 4y_{n-1} + y_{n-2}]$$

Combining all these expressions, we obtain

$$\int_{x_n}^{x_n} y\ dx = (h/3)[y_0 + 4(y_1 + y_3 + \ldots + y_{n-1}) +$$
$$+ 2(y_2 + y_4 + \ldots + y_{n-2}) + y_n]$$

This is known as Simpson's $1/3$ rule.

2.2.1 Example

Calculate the integral of the function, $f(x) = 2x$ in the interval $(0,1)$ using Simpson's $1/3$ rule.

Let $h = 1/3$. The tabulated values of x and y are given below

x	0	1/6	1/3	½	2/3	5/6	1
$y = f(x)$	0	1/3	2/3	1	4/3	5/3	2

Using Simpson's 1/3 Rule

$$I = \left(\frac{1}{18}\right)\left[0 + 4\left(\frac{1}{3} + 1 + \frac{5}{3}\right) + 2\left(\frac{2}{3} + \frac{4}{3}\right) + 2\right] = 1$$

2.3 Simpson's 3/8 Rule

Putting $n = 3$ in (2) all differences higher than the third term will become zero and therefore we have

$$\int_{x_0}^{x_3} y\ dx = 3h[y_0 + \frac{3}{2}\Delta y_0 + \frac{9}{12}\Delta^2 y_0 + \frac{3}{24}\Delta^3 y_0]$$
$$= \frac{3}{8}h[y_0 + 3y_1 + 3y_2 + y_3]$$

Similarly, $\int_{x_3}^{x_6} y\ dx = \frac{3}{8}h[y_3 + 3y_4 + 3y_5 + y_6]$ and so on.

Summing up all these, we obtain [3]

$$\int_{x_0}^{x_n} y\ dx = (3h/8)[y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 +$$
$$+ 2y_6 + \ldots + 2y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n]$$

This is known as Simpson's $3/8$ rule.

2.3.1 Example

We solve the integral of the function, $f(x) = x^2 + 1$, in the interval $[1, 4]$ using Simpson's $3/8$ rule.

Let $h = 1$. The tabulated values of x and y are given below:

x	1	2	3	4
y	2	5	10	17

From Simpson's 3/8 Rule,

$$\int_{x_0}^{x_n} y\,dx = (3h/8)[y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + $$
$$+ 2y_6 + ... + 2y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n]$$

2.4 Weddle's Rule

Putting $n = 6$ in (2) all differences higher than the first will become zero and we obtain

$$\int_{x_0}^{x_6} y\,dx = \frac{3h}{10}[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6]$$

And thus the general form for Weddle's rule is

$$\int_{x_0}^{x_n} y\,dx = (3h/10)[y_0 + 2(y_6 + y_{12} + ... + y_{n-6}) + $$
$$+ 5(y_1 + y_7 + ... + y) + 6(y_3 + y_9 + ... + y_{n-3}) + y_n]$$

2.4.1 Example

We evaluate $\int_o^6 2x\,dx$ up to three decimal places, by Weddle's rule.

Let $h = 1.0$. The tabulated values of x and y are given below

x	0	1	2	3	4	5	6
y	0	2	4	6	8	10	12

From Weddle's Rule,

$$\int_{x_0}^{x_n} y\,dx = (3h/10)[y_0 + 5(y_1 + y_5) + 6y_3 + $$
$$+ (y_2 + y_4) + y_6] = 36$$

3. Algorithm

Now we are introducing an algorithm to evaluate an integral numerically by four well known methods namely Trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule and Weddle's rule simultaneously and it will ensure which method gives the best accuracy compare to the exact solution.

INPUT: function f, limits x_0 and x_n, number of division n, direct result r.

Step-1: Compute $h = \frac{x_n - x_0}{n}$

Step-2: Set $i = 0$

Step-3: While $i \le n$, repeat Step- 4

Step-4: Set $y[i] = f(x_0 + ih)$

Step-5: Set $i = 1$

Step-6: While $i < n$, repeat Step-7

Step-7: Set $sum\_1 = sum\_1 + 2y[i]$

If $i \% 2 = 0$

Set $sum\_2 = sum\_2 + 2y[i]$

Else

Set $sum\_2 = sum\_2 + 4y[i]$

If $i \% 3 = 0$

Set $sum\_3 = sum\_3 + 2y[i]$

Else

Set $sum\_3 = sum\_3 + 3y[i]$

$t\_Sum\_1 = (h/2) * (y[0] + y[n] + sum\_1)$

$t\_Sum\_2 = (h/3) * (y[0] + y[n] + sum\_2)$

$t\_Sum\_3 = (3h/8) * (y[0] + y[n] + sum\_3)$

If $i \% 6 = 0$

Set $sum\_4 = sum\_4 + 2y[i]$

Else if $i \% 3 = 0$

Set $sum\_4 = sum\_4 + 6y[i]$

Else

Set $sum\_4 = sum\_4 + 4y[i]$

Else If ($i \% 6 = 1 \| i \% 6 = 5$)

$sum\_4 = sum\_4 + 5y[i]$

Else

$sum\_4 = sum\_4 + y[i]$

$t\_sum\_4 = (3 * h/8) * (y[0] + y[n] + sum\_4)$

Step-8: Set $a = \left| t\_sum\_1 - r \right|$, $b = \left| t\_sum\_2 - r \right|$,

$c = |t\_sum\_3 - r|$, $d = |t\_sum\_4 - r|$

If ($a < b$ and $a < c$)

If ($a < d$)

t_Sum_1

Else

t_sum_4

If ($b < a$ and $b < c$)

If ($b < d$)

t_Sum_2

Else

t_sum_4

If ($c < a$ and $c < b$)

If ($c < d$)

t_Sum_3

Else

t_sum_4

OUTPUT: t_Sum_1, t_Sum_2, t_Sum_3,

t_sum_4 with message which sum is most accurate.

STOP.

4 Findings

We are now giving a trial of the simulator for the numerical integration problem $I = \int_0^1 \frac{1}{1+x} dx$.

4.1 Input:

This is a program to evaluate numerical integration and check which method gives the best accuracy.

Enter equation:-

$\frac{1}{1+x} = I$

Type the initial value & final value:

0

1

Type the number of division.

Please enter 6 or multiple of 6:

12

Type direct result of integration:

.693140

4.2 Output:

The Trapezoidal rule gives: 0.693581

The Simpson $1/3$ rule gives: 0.693149

The Simpson $3/8$ rule gives: 0.693150

The Weddle's rule gives: 0.693147

Weddle's rule gives the best among these.

5 Conclusion

In this paper, we develop a simulator incorporated with traditional Trapezoidal rule, Simpson's $1/3$ rule, Simpson's $3/8$ rule and Weddle's rule for solving numerical integration. We observed that the result obtained according to our procedure is completely identical with the hand calculation. We therefore, hope that this simulator can evaluate integrals numerically to save time and labour. Moreover, we have seen that Weddle's rule gives the best solution among the four methods.

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