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Accelerated Life Testing Design using Geometric Process for Marshall-Olkin Extended Exponential Distribution with Type II censored data

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Abstract— ***In this paper the geometric process is used for the analysis of accelerated life testing for Marshall-Olkin Extended Exponential (MOEE) distribution using Type II censored data. Assuming that the lifetimes under increasing stress levels form a geometric process, The parameters are estimated by using the maximum likelihood method and the original parameters instead of the developing inference for the parameters of the log linear link function are used. The asymptotic interval estimates of the parameters of the distribution using Fisher information matrix are also obtained. The simulation study is conducted to illustrate the statistical properties of the parameters and the confidence intervals.***

Keywords— Maximum Likelihood Estimation; Type II Censoring; Survival Function; Fisher Information Matrix; Asymptotic Confidence Interval; Simulation Study.

1. Introduction

Models and methods of accelerated life-testing are useful when technical systems under test tend to have long lifetimes. Under normal operating conditions, as systems usually last long, the corresponding life-tests become too time-consuming and expensive. In these cases, accelerated life tests (ALT) can be applied to reduce the experimental time and hence the cost. There are basically three types of accelerated life tests: constant-stress test, step-stress test and progressive stress.

In constant-stress test each experimental unit put at only one of the stress levels and in step-stress the level of stress is increased step by step until all items have failed or the test stops for other reasons. Progressive-stress loading is quite like the step stress testing with the difference that the stress level increases continuously

Failure data obtained from ALT can be divided into two categories: complete (all failure data are available) or censored (some of failure data are missing). Due to different types of censoring, censored data can be divided into time-censored (or type I censored) data and failure-censored (or type II censored) data. Time censored (or type I censored) data is usually obtained when censoring time is fixed, and then the number of failures in that fixed time is a random variable. Failure censored (or type II censored) data is obtained when the test is terminated after a specified number of failures, and then time to obtain that fixed number of failures is a random variable. For more details about ALTs one can consult Bagdonavicius and Nikulin [i], Meeker and Escobar [ii], Nelson [iii, iv], Mann and Singpurwalla [v].

Constant stress ALT with different types of data and test planning has been studied by many authors. Watkins and John [vi] considers constant stress accelerated life tests based on Weibull distributions with constant shape and a log-linear link between scale and the stress factor which is terminated by a Type-II censoring regime at one of the stress levels. Ding et al. [vii] dealt with Weibull distribution to obtain accelerated life test sampling plans under type I progressive interval censoring with random removals. Pan et al. [viii] proposed a bivariate constant stress accelerated degradation test model by assuming that the copula parameter is a function of the stress level that can be described by a logistic function. Chen et al. [ix] discuss the optimal design of multiple stresses constant accelerated life test plan on non-rectangle test region. Fan and Yu [x] discuss the reliability analysis of the constant stress accelerated life tests when a parameter in the generalized gamma lifetime distribution is linear in the stress level. Islam and Ahmad [xi] and Ahmad and Islam [xii] discuss the optimal constant stress accelerated life test designs under periodic inspection and Type-I censoring. Geometric process (GP) is first used by Lam [xiii] in the study of repair replacement problem. Kamal [xiv] estimates Weibull parameters in accelerated life testing using geometric process with type-II censored data.

This article is to focus on the maximum likelihood method for estimating the acceleration factor and the parameters of Marshall-Olkin Extended distribution. This work was conducted for CSALT with type II censored scheme. The confidence intervals for parameters are also obtained by using the asymptotic properties of normal distribution. In the last, the statistical properties of estimates and confidence intervals are examined through a simulation study.

2. Model Description

2.1 The geometric Process

A GP is a stochastic process $\{X_n, n=1,2,...\}$ such that $\{\lambda^{n-1}X_n, n=1,2,...\}$ forms a renewal process where $\lambda>0$ is real valued and called the ratio of the GP. It is easy to show that if $\{X_n, n=1,2,...\}$ is a GP and the probability density function of X_1 is $f(x)$ with mean μ and variance σ^2 then the probability density function of X_n will be $\lambda^{n-1}f(\lambda^{n-1}x)$ with mean μ/λ^{n-1} and variance $\sigma^2/\lambda^{2(n-1)}$. It is clear to see that a GP is stochastically increasing if $0<\lambda<1$ and stochastically decreasing if $\lambda>1$. Therefore, GP is a natural approach to analyze the data from a series of events with trend.

2.2 The Marshall-Olkin Extended Exponential distribution (MOEE)

Marshall and Olkin [xv] proposed a new method for adding a parameter to a family of distributions. Suppose we have a given distributionwith survival function (SF) $\overline{F}(x), -\infty<x<\infty$ then the Marshall-Olkin extended distribution is defined by the SF

$$\bar{G}(x) = \frac{\alpha \bar{F}(x)}{1 - \bar{\alpha}\bar{F}(x)} \qquad -\infty < x < \infty, \alpha > 0, \bar{\alpha} = 1-\alpha \quad (1)$$

If the survival function of exponential distribution is $\bar{F}(x) = e^{-\theta x}, x, \theta > 0$, and put it in equation (1), we obtain the SF

$$\bar{G}(x) = \frac{1-\bar{\alpha}}{e^{\theta x} - \bar{\alpha}}, \qquad -\infty < x < \infty, \alpha > 0, \bar{\alpha} = 1-\alpha \quad (2)$$

The distribution with the survival function (2) is called the MOEE with parameters α and θ. The probability density function (pdf) and the cumulative distribution function (cdf) and the hazard rate of the Marshall-Olkin extended exponential with the survival function (2), respectively are given by

$$g(x;\alpha,\theta) = \frac{\alpha\theta e^{\theta x}}{[e^{\theta x} - \bar{\alpha}]^2}, \qquad -\infty < x < \infty, \alpha > 0, \bar{\alpha} = 1-\alpha \quad (3)$$

$$G(x;\alpha,\theta) = \frac{e^{\theta x} - 1}{[e^{\theta x} - \bar{\alpha}]} \quad (4)$$

$$r(x) = \frac{\theta e^{\theta x}}{[e^{\theta x} - \bar{\alpha}]} \quad (5)$$

when $\alpha = 1$, the pdf, cdf, SF and hazard rate reduce to those of the exponential distribution.

Assumptions

- Let there are s increasing stress levels and under each stress level n items are inspected. Let the random variable $x_{k(i)}$ denote the lifetime of the tested product, where $k, k = 1,2,.......s$ index for stress level and $i, i = 1,2........n$ is the index for test item under each stress. In the Type II censoring scheme, the test is terminated at each stress level when r failures are observed. Under the k^{th} stress level, only the r smallest observations of the sample will be collected, whose failure time can be written as $x_{k(1)} \le x_{k(2)} \le .... \le x_{k(r)}$.
- The product life follows a Marshall-Olkin Extended Exponential distribution given by (1) at any stress.
- Let the sequence of the random variables $X_0, X_1, X_2, ..., X_n$, denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily. We assume $\{X_k, k = 1,2,3,.....s\}$ is a geometric process with ratio $\lambda > 0$.
- At any constant stress level S_k, the mean life $1/\theta_k$ is a log linear function of stress, that is, $\log(1/\theta_k) = a + bS_k$, where a and b are unknown parameters that depend on the nature of the product and the test method. When $k = 0$, the above equation depicts the relation of the mean life and the designed stress level.

2.3 G.P assuption in ALT

From the above assumption, it can be easily shown that

$$\log\left(\frac{\theta_k}{\theta_{k+1}}\right) = b(S_{k+1} - S_k) = b\Delta S$$

Above relation can be written a

$$\left(\frac{\theta_{k+1}}{\theta_k}\right) = e^{-b\Delta S}$$

It tells that when the increased stress levels form an arithmetic sequence with a constant difference ΔS, the mean life under each stress level forms a geometric sequence with the ratio $e^{-b\Delta S}$.

Let $e^{-b\Delta S} = \lambda$, it is clear from the above relation

$$\theta_k = \lambda\theta_{k-1} = \lambda^2\theta_{k-2} = ... = \lambda^k\theta$$

In case of MOEE distribution, the pdf of a test item at the k^{th} stress level is:

$$g_{X_k}(x|\alpha,\theta,\lambda) = \frac{\alpha\theta_k e^{\theta_k x}}{[e^{\theta_k x} - \bar{\alpha}]^2} = \frac{\alpha\lambda\theta_{k-1} e^{\lambda\theta_{k-1} x}}{[e^{\lambda\theta_{k-1} x} - \bar{\alpha}]^2} = \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x}}{(e^{\theta\lambda^k x} - \bar{\alpha})^2}$$

This implies that

$$g_{X_k}(x) = \lambda^k g_{x_0}(\lambda^k x)$$

Therefore the pdf of a test item at the k^{th} stress level is

$$g_{X_k}(x) = \lambda^k g(\lambda^k x) = \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x}}{(e^{\theta\lambda^k x} - \bar{\alpha})^2} \quad (6)$$

Consequently, the CDF of the test item at the k^{th} stress level is:

$$G_{X_k}(x \mid \alpha,\theta,\lambda) = \frac{e^{\theta\lambda^k x} - 1}{e^{\theta\lambda^k x} - \bar{\alpha}} \quad (7)$$

The probability that an item censored at time $x_{k(r)}$ is:

$$\bar{G}_{X_{k(r)}}(x) = \frac{1-\bar{\alpha}}{e^{\theta\lambda^k x_{kr}} - \bar{\alpha}} \quad (8)$$

For those r_k items that fail at time t, their order statistics can be denoted by $X_{k(i)}$ with PDF:

$$g_{X_{K(i)}}(x) = \frac{n!}{(i-1)!(n-i)!} g_{X_{k(i)}} \left[G_{X_k}(x)\right]^{i-1} \left[1 - G_{X_k}(x)\right]^{n-i},$$
$$i = 1,2,........,r. \quad (9)$$

3. Maximum Likelihood Estimation

Let the test terminates at r^{th} failure time $x_{k(r)}$ and $(n-r)$ units are still surviving at each stress level. The likelihood function of the k^{th} stress can be expressed as above

$$L_k = \frac{n!}{(n-r)!} g_X\left(x_{k(1)}\right)................g_{X_k}\left(x_{k(r)}\right)\left[\bar{G}_{X_k}\left(x_{k(r)}\right)\right]^{n-r}$$

(10)

We derive the ML estimates of $\alpha, \theta,$ and λ from the likelihood function given by (6). The substitution of (6) and (8) in (10) gives

$$L_k = \frac{n!}{(n-r)!}\left(\prod_{i=1}^{r} \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x_{ki}}}{\left[e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right]^2}\right)\left(\frac{\alpha}{e^{\theta\lambda^k x_{kr}} + \alpha - 1}\right)^{n-r}$$

$$, 0 \le x_{k(1)} \le \dots\dots\dots \le x_{k(r)} \qquad (11)$$

It follows that the likelihood function of observed data in a total s stress levels is:

$$L = \prod_{k=1}^{s}\left[\frac{n!}{(n-r)!}\left(\prod_{i=1}^{r} \lambda^k \frac{\alpha\theta e^{\theta\lambda^k x_{ki}}}{\left[e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right]^2}\right)\left(\frac{\alpha}{e^{\theta\lambda^k x_{kr}} + \alpha - 1}\right)^{n-r}\right]$$

$$0 \le x_{k(1)} \le \dots\dots\dots\dots\dots \le x_{k(r)} ; 1 \le k \le s. \quad (12)$$

The log-likelihood function corresponding (12) can be rewritten as

$$l = \sum_{k=1}^{s}\left(\ln\frac{n!}{(n-r)!}\right) + \sum_{k=1}^{s}\sum_{i=1}^{r}\left[k\ln\lambda + \ln\alpha + \ln\theta + \theta\lambda^k x_{ki} - 2\ln\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)\right]$$
$$+ \sum_{k=1}^{s}(n-r)\left[\ln\alpha - \ln\left(e^{\theta\lambda^k x_{kr}} + \alpha - 1\right)\right]$$

MLE's of α, θ and λ are obtained by solving the equations $\frac{\partial l}{\partial\alpha} = 0, \frac{\partial l}{\partial\theta} = 0$ and $\frac{\partial l}{\partial\lambda} = 0$, where

$$\frac{\partial l}{\partial\alpha} = \frac{ns}{\alpha} - 2\sum_{k=1}^{s}\sum_{i=1}^{r}\frac{1}{\left(e^{\theta\lambda^k x_{ki}} + \alpha - 1\right)} - \sum_{k=1}^{s}(n-r)\frac{1}{\left(e^{\theta\lambda^k x_{kr}} + \alpha - 1\right)}$$

(13)

$$\frac{\partial l}{\partial\theta} = \frac{rs}{\theta} + \sum_{k=1}^{s}\sum_{i=1}^{r}\lambda^k x_{ki} - 2\sum_{k=1}^{s}\sum_{i=1}^{r}\frac{\lambda^k x_{ki} e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)}$$
$$- \sum_{k=1}^{s}\frac{(n-r)\lambda^k x_{kr} e^{\theta\lambda^k x_{kr}}}{\left(e^{\theta\lambda^k x_{kr}} + \alpha - 1\right)} \qquad (14)$$

$$\frac{\partial l}{\partial\lambda} = \sum_{k=1}^{s}\sum_{i=1}^{r}\frac{k}{\lambda} + \theta\sum_{k=1}^{s}\sum_{i=1}^{r}k\lambda^{k-1}x_{ki} - 2\theta\sum_{k=1}^{s}\sum_{i=1}^{r}\frac{k\lambda^{k-1}x_{ki}e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)}$$
$$- \sum_{k=1}^{s}\frac{(n-r)\theta k\lambda^{k-1}x_{kr}e^{\theta\lambda^k x_{kr}}}{\left(e^{\theta\lambda^k x_{kr}} + \alpha - 1\right)} \qquad (15)$$

The MLEs of $\alpha, \theta, and\, \lambda$ exist but do not have a closed form. The Newton Iterative Method is applied to obtain $\hat{\alpha}, \hat{\theta}, and\, \hat{\lambda}$.

4. Fisher Information Matrix

The Fisher information matrix F is obtained by taking the negative second partial derivatives of the log-likelihood function and can be written as

$$F = \begin{bmatrix} -\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\theta} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda} \\ -\frac{\partial^2 l}{\partial\theta\partial\alpha} & -\frac{\partial^2 l}{\partial\theta^2} & -\frac{\partial^2 l}{\partial\theta\partial\lambda} \\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda\partial\theta} & -\frac{\partial l}{\partial\lambda^2} \end{bmatrix}$$

Elements of Fisher Information matrix are

$$\frac{\partial^2 l}{\partial\alpha^2} = -\frac{ns}{\alpha^2} + 2\sum_{k=1}^{s}\sum_{i=1}^{r}\frac{1}{\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)^2} + \sum_{k=1}^{s}\frac{(n-r)}{\left(e^{\theta\lambda^k x_{kr}} + \alpha - 1\right)^2}$$

$$\frac{\partial^2 l}{\partial\theta^2} = -\frac{rs}{\theta^2} - 2\sum_{k=1}^{s}\sum_{i=1}^{r}\left[\frac{(\alpha-1)\lambda^{2k}x_{ki}^2 e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)^2}\right] - \sum_{k=1}^{s}\left[\frac{(n-r)(\alpha-1)\lambda^{2k}x_{kr}{}^2 e^{\theta\lambda^k x_{kr}}}{\left(e^{\theta\lambda^k x_{kr}} + \alpha - 1\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\lambda^2} = -\sum_{k=1}^{s}\sum_{i=1}^{r}\frac{k}{\lambda^2} + \theta\sum_{k=1}^{s}\sum_{i-1}^{r}k(k-1)\lambda^{k-2}x_{ki}$$
$$-2\sum_{k=1}^{s}\sum_{i=1}^{r}\left[\frac{\theta k(k-1)\lambda^{k-2}x_{ki}e^{2\theta\lambda^k x_{ki}} + (\alpha-1)\left\{\theta k(k-1)\lambda^{k-2}x_{ki}e^{\theta\lambda^k x_{ki}} + \theta^2 k^2 \lambda^{2k-2}x_{ki}^2 e^{\theta\lambda^k x_{ki}}\right\}}{\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)^2}\right]$$

$$-\sum_{k=1}^{s}(n-r)\left[\frac{\theta k(k-1)\lambda^{k-2}x_{kr}e^{2\theta\lambda^k x_{kri}} + (\alpha-1)\left\{\theta k(k-1)\lambda^{k-2}x_{kr}e^{\theta\lambda^k x_{kr}} + \theta^2 k^2 \lambda^{2k-2}x_{kr}^2 e^{\theta\lambda^k x_{kr}}\right\}}{\left(e^{\theta\lambda^k x_{kr}} - 1 + \alpha\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\alpha\partial\theta} = \frac{\partial^2 l}{\partial\theta\partial\alpha} = 2\sum_{k=1}^{s}\sum_{i=1}^{r}\left[\frac{\lambda^k x_{ki}e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)^2}\right] + \sum_{k=1}^{s}\left[\frac{(n-r)\lambda^k x_{kr}e^{\theta\lambda^k x_{kr}}}{\left(e^{\theta\lambda^k x_{kr}} - 1 + \alpha\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\alpha\partial\lambda} = \frac{\partial^2 l}{\partial\lambda\partial\alpha} = 2\sum_{k=1}^{s}\sum_{i=1}^{r}\left[\frac{\theta k\lambda^{k-1}x_{ki}e^{\theta\lambda^k x_{ki}}}{\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)^2}\right] + \sum_{k=1}^{s}\left[\frac{\theta k\lambda^{k-1}x_{kr}e^{\theta\lambda^k x_{kr}}}{\left(e^{\theta\lambda^k x_{kr}} - 1 + \alpha\right)^2}\right]$$

$$\frac{\partial^2 l}{\partial\theta\partial\lambda} = \frac{\partial^2 l}{\partial\lambda\partial\theta} = \sum_{k=1}^{s}\sum_{i=1}^{r}k\lambda^{k-1}x_{ki}$$
$$-2\sum_{k=1}^{s}\sum_{i=1}^{r}\left[\frac{k\lambda^{k-1}x_{ki}e^{2\theta\lambda^k x_{ki}} + (\alpha-1)\{k\lambda^{k-1}x_{ki}e^{\theta\lambda^k x_{ki}} + \theta k\lambda^{2k-1}x_{ki}^2 e^{\theta\lambda^k x_{ki}}\}}{\left(e^{\theta\lambda^k x_{ki}} - 1 + \alpha\right)^2}\right]$$

$$-\sum_{k=1}^{s}(n-r)\left[\frac{k\lambda^{k-1}x_{kr}e^{2\theta\lambda^k x_{kr}} + (\alpha-1)\{k\lambda^{k-1}x_{kr}e^{\theta\lambda^k x_{kr}} + \theta k\lambda^{2k-1}x_{kr}^2 e^{\theta\lambda^k x_{kr}}\}}{\left(e^{\theta\lambda^k x_{kr}} - 1 + \alpha\right)^2}\right]$$

5. Asymptotic confidence interval estimates

Now the variance and covariance matrix of parameters can be written as

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$$\sum = \begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \theta} & -\frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 l}{\partial \theta \partial \alpha} & -\frac{\partial^2 l}{\partial \theta^2} & -\frac{\partial^2 l}{\partial \theta \partial \lambda} \\ -\frac{\partial^2 l}{\partial \lambda \partial \alpha} & -\frac{\partial^2 l}{\partial \lambda \partial \theta} & -\frac{\partial l}{\partial \lambda^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{\theta}) & ACov(\hat{\alpha}\hat{\lambda}) \\ ACov(\hat{\theta}\hat{\alpha}) & AVar(\hat{\theta}) & ACov(\hat{\theta}\hat{\lambda}) \\ ACov(\hat{\lambda}\hat{\alpha}) & ACov(\hat{\lambda}\hat{\theta}) & AVar(\hat{\lambda}) \end{bmatrix}$$

The $100(1-\beta)\%$ asymptotic confidence interval for α,θ and λ are then given respectively as

$$\left[\hat{\alpha} \pm Z_{1-\frac{\beta}{2}}\sqrt{AVar(\hat{\alpha})}\right], \left[\hat{\theta} \pm Z_{1-\frac{\beta}{2}}\sqrt{AVar(\hat{\theta})}\right] \text{ and}$$

$$\left[\hat{\lambda} \pm Z_{1-\frac{\beta}{2}}\sqrt{AVar(\hat{\lambda})}\right]$$

6. Simulation Study

The performance of the estimates can be evaluated some measures of accuracy which are the mean squared error (MSE) and the coverage rate of asymptotic confidence intervals for different sample sizes and stress levels. The main purpose of simulation is to obtain MLEs of estimator and judge their performance for this model.

- To perform the simulation study first a random sample $x_{ki}, k = 1,2,....s, i = 1,2....r$ is generated from MOEE distribution which is censored at $r = 30,35$.
- The value of the parameters and numbers of the stress levels are chosen to be $\theta = 0.2$, $\alpha = 0.9$, $\lambda = 4$ and $s = 2,4$.
- For different sample sizes n= 40, 60, 80,100 & 200 ML estimates, the mean squared error (MSE), lower and upper asymptotic confidence interval limits and the coverage rate of the 95% confidence interval of parameters are obtained. The results obtained in the above simulation study are summarized in Table 1, 2, 3 & 4.

Table I
Simulations Results Based on Censored Data from GP MOEE with θ=0.2, α =0.9 and λ =4 for s=2 & r=30

n	$\hat{\theta}$ $\hat{\alpha}$ $\hat{\lambda}$	MSE $(\hat{\theta})$ MSE $(\hat{\alpha})$ MSE $(\hat{\lambda})$	95% Confidence Interval		95% Confidence Interval Coverage
			LCL	UCL	
40	1.1134 0.0524 1.1426	0.3213 0.0464 0.7464	1.1577 0.0504 1.1345	1.1456 0.0634 1.1789	0.94342 0.94569 0.94433
60	1.1078 0.0456 1.0565	0.2345 0.0345 0.6234	1.1096 0.0367 1.0455	1.2535 0.0486 1.0765	0.95896 0.96789 0.95662
80	1.1024 0.0367 1.0534	0.2245 0.0312 0.4424	1.1018 0.0254 1.0341	1.1065 0.0465 1.0655	0.95644 0.96547 0.95778
100	1.0234 0.0354 1.0134	0.2145 0.0234 0.4134	1.0214 0.0343 1.0123	1.0356 0.0455 1.0256	0.96654 0.97986 0.98766
200	1.0154 0.0165 1.0121	0.1575 0.0187 0.3567	1.0145 0.0157 1.0112	1.0167 0.0169 1.0322	0.96775 0.97997 0.98979

Table 2
Simulations Results Based on Censored Data from GP MOEE with θ=0.2, α =0.9 and λ =4 for s=2 & r=35

n	$\hat{\theta}$ $\hat{\alpha}$ $\hat{\lambda}$	MSE $(\hat{\theta})$ MSE $(\hat{\alpha})$ MSE $(\hat{\lambda})$	95% Confidence Interval		95% Confidence Interval Coverage
			LCL	UCL	
40	1.2811 0.0635 1.5734	0.5731 0.0774 1.3837	1.1179 0.0573 1.2772	1.3645 0.0874 1.6677	0.93831 0.92984 0.93784
60	1.2534 0.0624 1.5646	0.5717 0.0676 1.2847	1.1763 0.0563 1.3636	1.3771 0.0776 1.7731	0.93736 0.95341 0.93857
80	1.1837 0.0731 1.5663	0.3733 0.0635 1.2841	1.1254 0.0582 1.4883	1.2764 0.0767 1.6554	0.95736 0.93836 0.93726
100	1.1532 0.0701 1.4525	0.2736 0.0573 1.1836	1.1235 0.0563 1.3746	1.3746 0.0781 1.6831	0.95837 0.94736 0.95836
200	1.0328 0.0572 1.4281	0.4786 0.0463 1.1577	1.0273 0.0471 1.3745	1.2774 0.0589 1.7663	0.95732 0.96722 0.96721

Table 3
Simulations Results Based on Censored Data from GP MOEE with θ=0.2, α =0.9 and λ =4 for s=4 & r=30

n	$\hat{\theta}$ $\hat{\alpha}$ $\hat{\lambda}$	MSE $(\hat{\theta})$ MSE $(\hat{\alpha})$ MSE $(\hat{\lambda})$	95% Confidence Interval		95% Confidence Interval Coverage
			LCL	UCL	
40	1.1255 0.0543 1.6445	0.2654 0.0656 0.2997	1.1087 0.0503 1.6043	1.2787 0.0697 1.7675	0.93567 0.93786 0.94886
60	1.1144 0.0524 1.5770	0.2575 0.0624 0.2769	1.1129 0.0487 1.3458	1.1209 0.0579 1.7807	0.93864 0.94785 0.94324
80	1.0865 0.0475 1.4764	0.2522 0.0586 0.2345	1.0346 0.0267 1.3467	1.2543 0.0586 1.5578	0.95464 0.94456 0.94342
100	1.0531 0.0374 1.3664	0.2235 0.0478 0.2257	1.0154 0.0235 1.1686	1.0757 0.0389 1.4689	0.95888 0.95890 0.96868
200	1.0134 0.0167 1.2565	0.1755 0.0165 0.2142	1.0127 0.0158 1.0458	1.0174 0.0198 1.2896	0.96454 0.97675 0.97865

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Table 4
Simulations Results Based on Censored Data from GP MOEE with θ=0.2, α =0.9 and λ =4 for s=4 & r=35

n	$\hat{\theta}$ $\hat{\alpha}$ $\hat{\lambda}$	MSE $(\hat{\theta})$ MSE $(\hat{\alpha})$ MSE $(\hat{\lambda})$	95% Confidence Interval		95% Confidence Interval Coverage
			LCL	UCL	
40	1.1134 0.0524 1.1426	0.3213 0.0464 0.7464	1.1577 0.0504 1.1354	1.1456 0.0634 1.1789	0.94342 0.94569 0.94433
60	1.1078 0.0456 1.0565	0.2345 0.0345 0.6234	1.1096 0.0367 1.0455	1.2535 0.0486 1.0765	0.95896 0.96789 0.95662
80	1.1024 0.0367 1.0534	0.2245 0.0132 0.4424	1.1018 0.0254 1.0341	1.1065 0.0465 1.0655	0.95644 0.96547 0.95778
100	1.0234 0.0354 1.0134	0.2145 0.0235 0.4143	1.0214 0.0243 1.0123	1.0356 0.0455 1.0256	0.96654 0.97986 0.98766
200	1.0154 0.0165 1.0121	0.1575 0.0187 0.3567	1.0145 0.0157 1.0112	1.0167 0.0169 1.0322	0.96775 0.97997 0.98979

7. Conclusions

From the results in above tables, it is easy to find that estimates of the parameter perform well. For fixed θ, α and λ, the MSEs decreases as *n* increases and the probability coverage is also satisfactory. This indicates that the ML estimates provide asymptotically normally distributed and consistent estimator for the parameters. For the fixed sample sizes, as number of failures *r* gets larger the MSEs of the estimators decrease. This is very usual because more failures increase the efficiency of the estimators.

From above discussion and results it is concluded that the present model work well under type-II censored data for MOEE distribution and would be a good choice to be considered in the field of ALTs in future. For the further research in this direction one can choose some other lifetime distribution with different types of censoring schemes.

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