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Some Unusual Observations about an Achromatic Index of Graphs

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Abstract:* : *This paper addresses some interesting observations about an achromatic index of graphs related to degree sequence of graphs, graphs & their complements & existing achromatic index of graphs.

Keywords : graphs, achromatic, colouring

Introduction:

A k-edge colouring of a simple graph G is assigning k colours to the edges of G so that no two adjacent edges receive same colours. If for each pair t_i & t_j of colours, there exist adjacent edges with these colours then the colouring is said to be complete. Let G be a simple graph. The achromatic index $\psi^{|}$ (G) of a simple graph G is the maximum number of colours used in the edge colouring of G such that the colouring is complete. All though $\psi^{|}$(G) is known for some graphs but in general it is not known for arbitrary simple graphs. For Complete graph G of order n, $\psi^{|}$ (G) is denoted either by $A(K_n)$ or A(n).

Key words: **Achromatic index, colouring of graphs, complete edge colouring , degree sequnce**

Let us begin with the following observations.

Observation 1:

It is known that for a simple graph G(p, q) whose degree sequence is $d_1\ d_2\ d_3\ldots\ldots d_p$

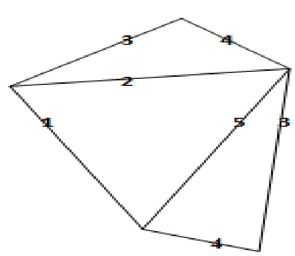
$$\psi^{|}(G)\,(\psi^{|}(G)-1) \le \left(\sum {}^{d_i}P_2\right)$$

where the sum is taken over i=1to p [2]

So it is natural to expect higher the number $\sum {}^{d_i}P_2$ gives higher Achromatic Index of G

But this expectation is not true in general.

Consider the following graphs G_1



(G_1)

(Numbers written on the edges are colours throughout this paper). It can be easily observe that the above colouring is the complete colouring of G_1

$\therefore\ \psi^{|}(G_1) \ge 5$ ……………*1

The degree sequence of G_1 is 2, 2, 2, 3, 4 hence

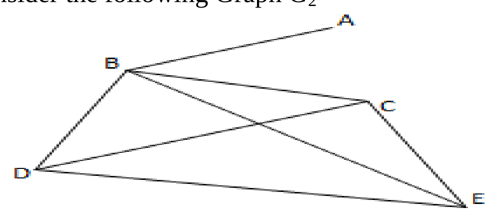
$$\sum {}^{d_i}P_2 = 28$$

$\therefore\ \psi^{|}(G_1)(\psi^{|}(G_1)-1) \le 28$

$\therefore\ \psi^{|}(G_1) \le 5$ …………....*2

$\therefore$ from *1 & *2 we conclude $\psi^{|}(G_1) = 5$

Now consider the following Graph G_2



(G_2)

The degree sequence of G_2 is 1, 3, 3, 3, 4 hence

$$\sum {}^{d_i}P_2 = 30$$

$\therefore\ \psi^{|}(G_2)\,(\psi^{|}(G_2)-1) \le 30$

$\therefore\ \psi^{|}(G_2) \le 6$

Now we will try to obtain a complete colouring of G_2 with five colours.

WLG BA=4 (we mean BA is coloured with the colour 4)
BE=3, BD=2, BC=1

Now we will think of inserting colour 5 in G_2

If CE=5 then the colour 2 cannot be made adjacent to the colour 5 as the choices for CD, DE will not establish the proper colouring of the graph G_2.

If CD=5 then the colour 3 cannot be made adjacent to the colour 5 as the choices for CE, DE will not establish the proper colouring of the graph G_2.

If DE=5 then the colour 1 cannot be made adjacent to the colour 5 as the choices for CD, CE will not establish the proper colouring of the graph G_2.

$\therefore\ \psi^{|}(G_2) \le 4$

The following colouring shows $\psi^{|}(G_2) = 4$

Though both graphs have same number of edges & same number of vertices & $\sum {}^{d_i}P_2$ of $G_1 \le \sum {}^{d_i}P_2$ of G_2 ,

$\psi^{|}(G_1) \ge \psi^{|}(G_2)$

Observation 2

Let H be a spanning sub graph of Kn. Then there may be any kind of inequality or equality between $\psi^{|}(H)+\psi^{|}(H^c)$ & $\psi^{|}(K_n)$ as shown in the following cases.

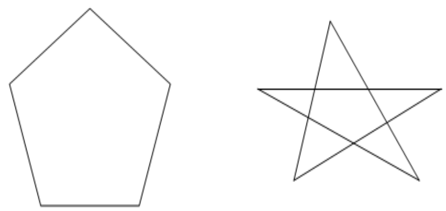
case i) Let n=11 H be K_{10} U {single isolated vertex} therefore H^c is a spanning$K_{1,10}$ tree of K_n

It is known that $\psi^{|}(K_n)=27$, $\psi^{|}(H)=10$, $\psi^{|}(H^c)=22$

$\therefore\ \psi^{|}(K_n) < \psi^{|}(H)+\psi^{|}(H^c)$

It is naturally posing like $A(n+1) \leq A(n) + n$ which is a trivial result.[1]

Case ii) Let n=5 & H, H^c are as shown below



(H) (H^c)

It is known that $\psi^{|}(K_5)=7$

It is easy to check $\psi^{|}(H)=3$, $\psi^{|}(H^c)=3$

$\therefore \psi^{|}(K_n) \geq \psi^{|}(H) + \psi^{|}(H^c)$

Case iii) let n=5

H be K_4 U {single isolated vertex} therefore H^c is a spanning $K_{1,4}$ tree of K_n

It is known that $\psi^{|}(K_n)=7$, $\psi^{|}(H)=3$, $\psi^{|}(H^c)=4$

$\therefore \psi^{|}(K_n) = \psi^{|}(H) + \psi^{|}(H^c)$

Observation 3:

The bounds of achromatic index of complete graphs K_n up to n=30 are given below.[1]

n	Lower Bound	Upper Bound
1	0	0
2	1	1
3	3	3
4	3	3
5	7	7
6	8	8
7	11	11
8	14	14
9	18	18
10	22	22
11	27	27
12	31	33
13	39	39
14	39	44
15	41	49
16	41	53
17	52	57
18	52	61
19	57	65
20	57	69
21	65	73
22	65	77
23	83	84
24	89	92
25	100	100
26	105	108
27	110	117
28	110	126
29	112	135
30	136	145

It can be observe that the condition

$$\mathbf{A(2n+1) \geq A(2n)[1 + 2n/(A(2n)+n(2n-1))]}$$

remains valid up to n=13, so if the condition is considered to be true then it tights some of the upper & lower bounds of achromatic indices of graphs.

For example substitute n=8 in the above condition

It gives $A(17) \geq A(16)[1+16/(A(16)+120)]$

$\therefore A(16) \leq A(17)/[1+16/(A(16)+120)]$

$\therefore A(16) \leq A(17)/[1+16/(41+120)]$

$\therefore A(16) \leq A(17)/1.09937888$

$\therefore A(16 \leq 51$

$\therefore$U.B. of K_{16} reduces to 51

If we substitute n=13

$A(27) \geq A(26)[1+26/(A(26)+325)]$

$\therefore A(27) \geq A(26)[1+26/(108+325)]$

$\therefore A(27) \geq 105 \text{x} 1.06$

$\therefore A(27) \geq 111$

$\therefore$L.B. of K_{27} rises to 111.

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