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Hyperchaos to Secure Communications According to Synchronization by a High Gain Observer

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Abstract - The purpose of this article is to secure the information message using a new six order continuous hyperchaotic system that we have developed. After studying and verifying the hyperchaotic behavior and stability of this system, a chaotic masking scheme is applied to secure the information between a transmitter and a receiver.
The results of the simulations confirm the high performance of the observer designed for this high order system and the proposed method leads to an almost perfect restoration of the original signal.

Keyword: 7D sixorder hyperchaotic system, Equilibrium point,Lyapunov exponent,High gain observer, Chaotic masking scheme.

I. INTRODUCTION

Over the past few years, chaos has been increasingly used to secure communications. Compared to other methods, the additive encryption method has advantages such as good security, high dynamic storage capacity and low power, which considerably improves the security and reliability of the transmission of information. When the signal from the chaotic system is used as an encrypted signal, the message can be decrypted and attacked easily. However, the characteristics of the hyperchaotic system are more complex. Thus, for greater safety, the use of the hyperchaotic signal presents a wider application perspective [1-4]. In [5-6], we give the four dimensional system, the six dimensional system and their construction circuits, which is the basis of the construction of the hyperchaotic system of superior dimension.
The objective of this article is to secure communications by using a new 7D hyperchaotic system and based on the theory of high gain observers. In this approach, once the drive system is given, the response system can be selected as an observer and the control signal must be selected so that the drive system verifies certain conditions.

II. RESEARCH METHOD

Consider the hyperchaotic system described by the dynamic:

$$\dot{x} = Ax + f(x) \qquad (1)$$

Where $x \in R^n$ is the state of the system, A is the $n\times n$matrix of the system parameters and$f: R^n \to R^n$ is the nonlinear part of the system.
We consider the system (1) as the drive system.
As the response system, we consider the following hyperchaotic system described by the dynamic:

$$\dot{y} = By + g(y) + K(\theta) \qquad (2)$$

Where $y \in R^n$ the state of the system, B is the $n\times n$matrix of the system parameters and $g: R^n \to R^n$ is the nonlinear part of the system and θ is the gain of the response system.
In the nonlinear feedback control approach, we design an observer, which synchronizes the states of the drive system (1) and the response system (2) for all initial conditions $(x(0), y(0)) \in R^n$.
If we define the synchronization error as:

$$e = y - x \qquad (3)$$

Thus, the synchronization problem is essentially to find a controller θ so as to stabilize the error e for all initial conditions $e(0) \in R^n$.
In our case θ is a high gain observer.

III. HIGH GAIN OBSERVER DESIGN

The problem of observer design naturally arises in a system approach, as soon as one needs unmeasured internal information from external measurements. As we know, it is almost impossible to measure all the elements of the state vector in practice (e.g., the unknown state variables, fault signals, etc).
State observers are able to provide a continuous estimation of some signals which are not measured by hardware sensors. They need a mathematical model of the process and hardware measurements of some other signals. An observer is a dynamic system whose input includes the control u and the output y and whose output is an estimate of the state vector $\hat{x}$ as it's shown in figure1.

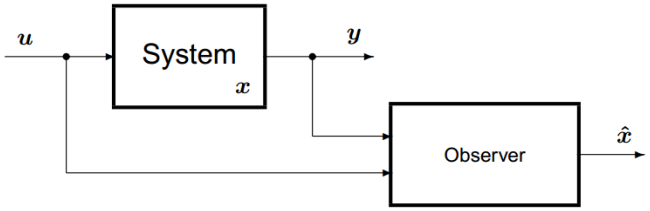

Fig1: Principle of the observer
We consider a general representation of the hyperchaotic system as follow:

$$\begin{cases} \dot{x} = f(x) \\ y = Cx \end{cases} \qquad (4)$$

Where:

- $C = (1\ 0\ \dots\ 0)$and$x \in R^n$ is the vector of states, $x = (x_1, x_2, \dots, x_n)^T$.
- $f: R^n \to R^n$is the nonlinear part of the system.
- $y \in R^m$ is the system measured output with $m < n$, and

$$A = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & & 1 & 0 \end{bmatrix}$$

It is necessary the design of an auxiliary system so called observer system to reconstruct then known states or unmeasurables. Firstly, we give necessary and sufficient conditions to establish whether the system (4) is observable.
Now, consider the following assumptions [5]:

A. Assumptions

A1. The system given in Eqs (4) is locally uniformly observable (Gauthier et al., 1992), hence for all $x \in R^n$, satisfies the observability rank condition:

$$rang\left\{\frac{\partial}{\partial x}\vartheta\right\} = n \qquad (5)$$

Here ϑ is the observability vector function define d$\vartheta = (dL_f^0 h, dL_f^1 h, \dots, dL_f^{n-1} h)^T L_f^r$ the r-order Lie derivatives, which are the directional derivatives of the corresponding state variables along the measured output trajectory. And $dL_f^r h$are the differentials of the r[th]-order Lie derivatives defined recursively as follows:

$$L_f^0 h := h, \quad dL_f^0 h := dh = \left(\frac{\partial h}{\partial x_1}, \dots, \frac{\partial h}{\partial x_n}\right)$$

$$L_f^1 h :=< dh, f >= \sum_{i=1}^{n} \frac{\partial h}{\partial x_i} f_i$$

$$dL_f^1 h := \left(\frac{\partial}{\partial x_1}\left(\sum_{i=1}^{n} \frac{\partial h}{\partial x_i} f_i\right), \dots, \frac{\partial}{\partial x_n}\left(\sum_{i=1}^{n} \frac{\partial h}{\partial x_i} f_i\right)\right)$$

$$L_f^r h :=< dL_f^{r-1} h, f >= L_f(L_f^{r-1} h), \qquad r \geq 2$$

A2. All the trajectories $x(t, x_0)$, $x_0 \in R^n$ of the system (4) are bounded.
Considering the set $\Omega\, c\, R^n$ as the corresponding physically realizable domain, such that:
$\Omega = \{(x_i)_{i=1}^n \in R_+^n / \ 0 \leq x_i \leq x_{max}\}$
In most practical cases, Ω will be an open connected relatively compact subset of R^n, and in the ideal cases, Ωwill be positively invariant under the dynamic (4).
In order to analyze the estimation error $\varepsilon = y - x$ we consider the next assumption.
A3. The nonlinear difference vector function $\Delta\emptyset = f(x) - f(\hat{x})$ is Lipschitz bounded i.e.$\Delta\emptyset < \Lambda|\varepsilon|$.
Condition A3 can be fulfilled satisfied if the following supremumis finite:
$\Lambda := sup_{x\in\Omega}\|f'(x)\|$
Where $f'(x)$ is the Jacobian and $\|.\|$ is the matrix norm associated with the Euclidian vector norm.

B. Propositions

For any initial condition $x(0) \in D$ and any $\hat{x}(0) \in D$and for θ large enough the system (1) satisfying previous assumptions [10] [11], can be estimated by the following dynamical system:

$$\dot{\hat{x}} = f(\hat{x}) - \theta * d_\theta^{-1} * S_1^{-1} * C'(C\hat{x} - y) \qquad (6)$$

Where S_1 is the symetric positive definite solution of the algebraic equation:
$\theta S_\theta + A'S_\theta + S_\theta A - C'C = 0$
For $\theta = 1$ and it can be expressed as
$S_1(i,j) = (-1)^{i+j} C_{i+j-2}^{j-1}$, for $1 \leq i,j \leq n$
where$C_j^i = \frac{j!}{i!(j-i)!}$
$d_\theta$is a diagonal matrix defined by :

$$d_\theta = diag\left(1, \frac{1}{\theta}, \dots, \frac{1}{\theta^n}\right)$$

C. Lemma

For θ large enough the slave system below is an observer for the drive system [13]:

$$\dot{\hat{x}} = f(\hat{x}) - \theta * d_\theta^{-1} * S_1^{-1} * C'(C\hat{x} - z) \qquad (7)$$

IV. SYSTEM DESCRIPTION

In this section we present the simulation results of the sixth order hyperchaotic system 7D is defined by [3]:

$$\begin{cases} \dot{x}_1 = 15\, x_1 + 20\, x_3 + 0.785\, x_2 x_3 x_4 x_5 x_6 x_7 \\ \dot{x}_2 = -20\, x_1 - 20\, x_2 - x_1 x_3 x_4 x_5 x_6 x_7 \\ \dot{x}_3 = 20\, x_4 - 15\, x_3 + 20\, x_5 + x_1 x_2 x_4 x_5 x_6 x_7 \\ \dot{x}_4 = 5\, x_4 - x_1 x_2 x_3 x_5 x_6 x_7 \\ \dot{x}_5 = -30\, x_5 + 19\, x_3 + x_1 x_2 x_3 x_4 x_6 x_7 \\ \dot{x}_6 = 10\, x_2 + 3.9\, x_6 - x_1 x_2 x_3 x_4 x_5 x_7 \\ \dot{x}_7 = 20\, x_3 - 15\, x_7 + 1.5\, x_1 x_2 x_3 x_4 x_5 z_6 \end{cases} \qquad (8)$$

The new hyperchaotic system (8) has unique equilibrium point$E_0(0,0,0,0,0,0,0)$.
Then the eigenvalues corresponding to equilibrium $E_0(0,0,0,0,0,0,0)$ will be obtained as follows:

$$\lambda_1 = 3.894, \lambda_2 = -14.927, \lambda_3 = -1.610,$$
$$\lambda_4 = 5.001, \lambda_5 = -14.943, \lambda_6 = -10.951,$$
$$\lambda_7 = -10.238$$

Where $\lambda_2,\ \lambda_3, \lambda_5,\ \lambda_6, \lambda_7$are negative real roots and $\lambda_1,\ \lambda_4$ are positive real roots.
Therefore, equilibrium $E_0$is unstable.
System (8) has three positive lyapunov exponents, which shows that the system is not only chaotic but also hyperchaotic.

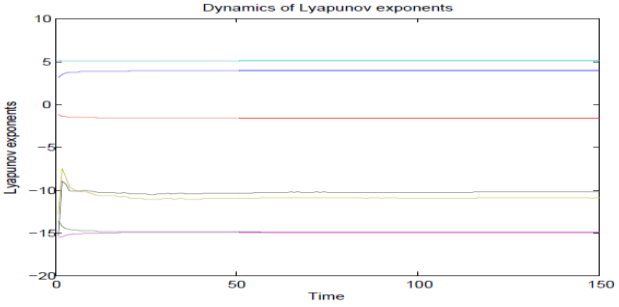

Fig2:Lyapunov exponent sevendimensional sixorder hyperchaotic system

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V. SYNCHRONIZATION PROBLEM FORMULATION

As reported in the literature, synchronization of hyperchaotic systems suggests the possibility for communication using hyperchaotic waveforms as carriers, perhaps with application to secure communication. The obvious approach uses a hyperchaotic system as the transmitter and a synchronous hyperchaotic system for the receiver, and several designs have been suggested that fit within this construct [8]. The variation in these designs lies in the methods for injecting an information signal at the transmitter and recovering it at the receiver.

Definition: Two hyperchaotic systems are completely synchronized if the error [9]:

$\|\hat{x} - x\| \to 0 \; as \; t \to \infty$

It means that each state in the slave system is identical or is very close to its corresponding state in the master system along the time.

Chaos synchronization as stabilization consists in finding a control command that leads the trajectories of a dynamical error system to the origin [14].

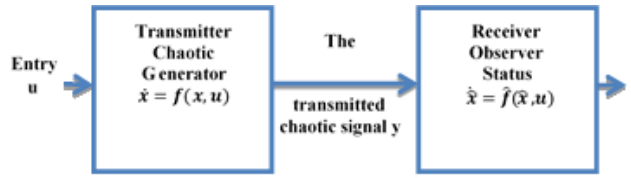

Fig3:Principle of Master Slave synchronization

In this section, we design a high observer design for the synchronization of our 7D hyperchaotic system.

The dynamic ofthe observer of our 7D sixorder hyperchaoticsystem is given by:

$$\begin{cases} \dot{\hat{x}}_1 = 15x_1 + 20\hat{x}_3 + 0.785\hat{x}_2\hat{x}_3\hat{x}_4\hat{x}_5\hat{x}_6\hat{x}_7 - \theta * d_\theta^{-1} * S_1^{-1} * C'C(\hat{x}_1 - x_1) \\ \dot{\hat{x}}_2 = -20\boldsymbol{s} - 20z_2 - \boldsymbol{s}\hat{x}_3\hat{x}_4\hat{x}_5\hat{x}_6\hat{x}_7 - \theta * d_\theta^{-1} * S_1^{-1} * C'C(\hat{x}_2 - x_2) \\ \dot{\hat{x}}_3 = 20\hat{x}_4 - 15\hat{x}_3 + 20\hat{x}_5 + \boldsymbol{s}\,\hat{x}_2\hat{x}_4\hat{x}_5\hat{x}_6\hat{x}_7 - \theta * d_\theta^{-1} * S_1^{-1} * C'C(\hat{x}_3 - x_3) \\ \dot{\hat{x}}_4 = 5\hat{x}_4 - \boldsymbol{s}\,\hat{x}_2\hat{x}_3\hat{x}_5\hat{x}_6\hat{x}_7 - \theta * d_\theta^{-1} * S_1^{-1} * C'C(\hat{x}_4 - x_4) \\ \dot{\hat{x}}_5 = -30\hat{x}_5 + 19\hat{x}_3 + \boldsymbol{s}\,\hat{x}_2\hat{x}_3\hat{x}_4\hat{x}_6\hat{x}_7 - \theta * d_\theta^{-1} * S_1^{-1} * C'C(\hat{x}_5 - x_5) \\ \dot{\hat{x}}_6 = 10\hat{x}_2 + 3.9\hat{x}_6 - \boldsymbol{s}\,\hat{x}_2\hat{x}_3\hat{x}_4\hat{x}_5\hat{x}_7 - \theta * d_\theta^{-1} * S_1^{-1} * C'C(\hat{x}_6 - x_6) \\ \dot{\hat{x}}_7 = 20\hat{x}_3 - 15\hat{x}_7 + 1.5\,\boldsymbol{s}\,\hat{x}_2\hat{x}_3\hat{x}_4\hat{x}_5\hat{x}_6 - \theta * d_\theta^{-1} * S_1^{-1} * C'C(\hat{x}_7 - x_7) \end{cases} \quad (9)$$

For the synchronization, the error e is defined as:

$e_1 = \hat{x}_1 - x_1, e_2 = \hat{x}_2 - x_2, \; e_3 = \hat{x}_3 - x_3, \; e_4 = \hat{x}_4 - x_4, \; e_5 = \hat{x}_5 - x_5, \; e_6 = \hat{x}_6 - x_6, \; e_7 = \hat{x}_7 - x_7 \quad (10)$

VI. CHAOTIC MASKING

This technique is considered the first proposal to use chaos to secure communication. It is presented in the references [7]. The principle is to scramble the message signal $m\,(t)$ in a chaotic signal $c\,(t)$ by a direct addition operation before transmitting it, to obtain an encrypted signal $s\,(t)$.

In order to recover the message signal at the authorized receiver, the same system generating the chaos is used both for transmission and reception, with the difference that in the receiver, this system is controlled by the received signal $r\,(t)$ (Equal to the signal $s\,(t)$ affected by the disturbances in the channel) to obtain the synchronization.

The order of magnitude of the message signal must be very low compared to that of the chaotic signal $c\,(t)$ as shown in figure 4 to give no hope of retrieving it by the intruders without knowing the signal $c\,(t)$And to have a good synchronization at the authorized receiver.

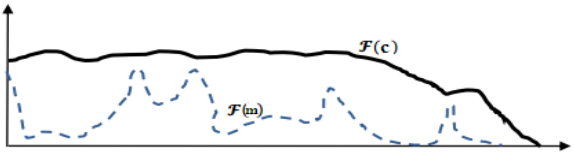

Fig 4: Spectrum of an information signal and chaotic signal

The encryption and decryption key is equal to the values of the coupling parameters between the transmitter and the receiver and the parameters characterizing the chaotic systems used.

Then the message signal is reconstituted by the difference between the received signal $r\,(t)$ and the signal $c\,(t)$ close to $c(t)$ result of the synchronization, see figure 5 which illustrates this principle.

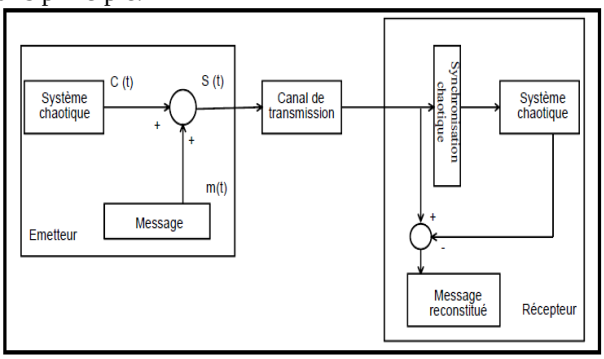

Fig 5: Schema of chaotic masking by addition

6.1. Mounting of masking and synchronization

In 1993, Cuomo proposed a much more interesting montage for this type of communication. Instead of transmitting two signals, it is possible to carry out an assembly with only the chaotic signal which will control the subsystem stable to the receiver. The signal will be masked by it with a small amplitude [8].

The subsystem was stable to the disturbances that would come from the information signal, it could be synchronized correctly. This assembly is illustrated in figure 6 where the stable subsystem is (y, z) and x is the chaotic control signal. On the other hand, this technique requires the subsystem x to be stable by being controlled by (y, z).

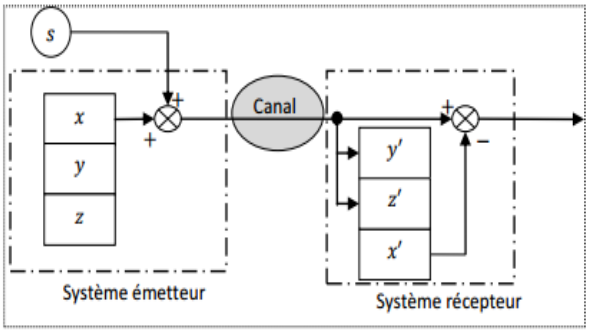

Fig 6: Mounting of Cuomo

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This arrangement has the advantage of revealing less information about the nature of the system (x, y, z) but the synchronization will obviously be less well so the recovered signal may be noisy. By choosing a sufficiently low amplitude ratio m / x and having an x which has a fairly large and regular spectrum, this system can be quite effective for securing information. Thus, the receiver system in our case is the high gain observer.

6.2. Simulation of hyperchaotic systems for additive masking

Thus, in this simulation, we use the synchronization between two identical systems that we verified in chapter three. In this way the first master system is taken as the generator of a chaotic signal $(c(t) = x_1(t))$ is a system state variable in the transmitter). At the receiver, the synchronization is done using the slave system (high gain observer) with the same initial conditions as those given in the transmitter, but controlled by the received signal $r\ (t\ )$.

Assuming that the perturbations due to the channel are negligible and that the transmission does not require modulation, we will then have:

$$r\,(t\,) = s\,(t\,) = c\,(t\,) + m(t\,)$$

from where :

$$s\,(t\,) = x_1(t\,) + m(t\,)$$

With $m(t\,)$ the message signal presented in figure 7.

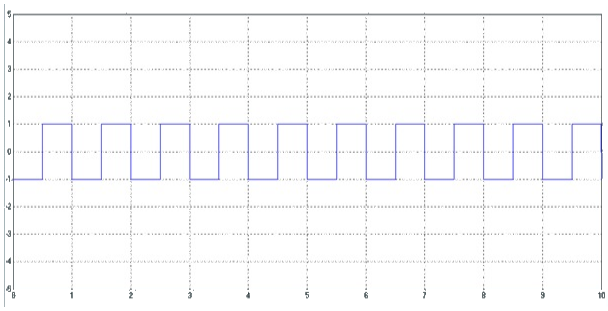

Fig7 : Original message m (t)

The encryption key (or decryption key) is the parameters of the sending system. A square signal with an amplitude equal to 1 and a frequency equal to 1 Hz is chosen for $m(t\,)$. At the transmitter, the encrypted message will then be received and at the output of the receiver, the reconstituted message will then be received.

VII. RESULTS AND ANALYSIS

The initial conditions of the7D sixorder hyperchaoticsystem(9) are chosen as:

$$x_1 = \widehat{x}_1 = 3,\ x_2 = \widehat{x}_2 = 1, x_3 = \widehat{x}_3 = -1, x_4 = \widehat{x}_4 = 1,$$
$$x_5 = \widehat{x}_5 = 2, x_6 = \widehat{x}_6 = -2,$$
$$x_7 = \widehat{x}_7 = 3$$

The simulation results for an observer gain $\theta\ =\ 5$ are as follows.

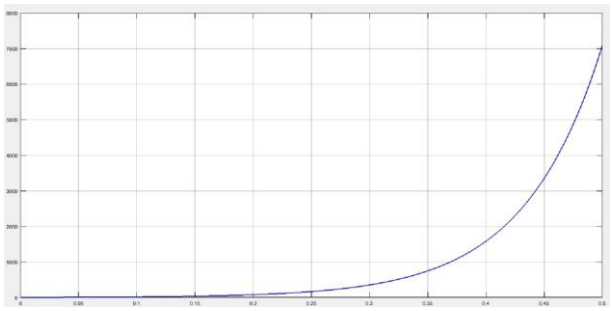

Fig 8: The state of the 7D six order hyperchaotic System

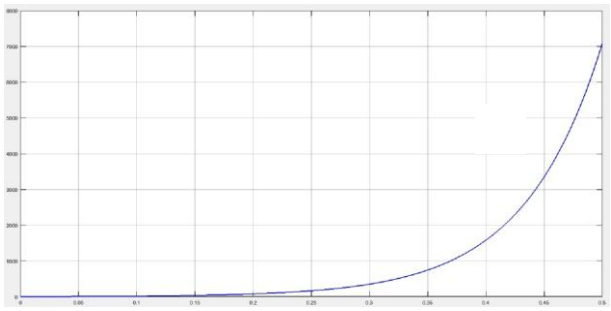

Fig 9:Encrypted message *s (t)* of the 7D six order hyperchaotic System

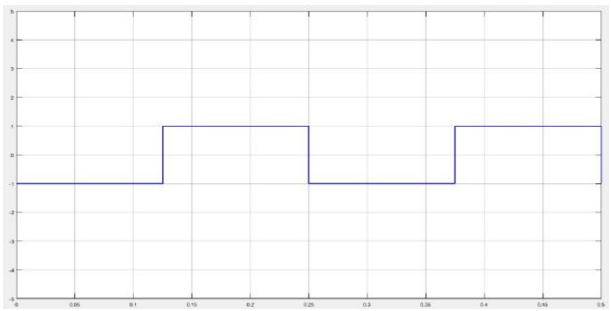

Fig 10: Restored message $m'(t\,)$ by subtracting the state $x_1(t\,)$ from the observer of the 7D system of six order

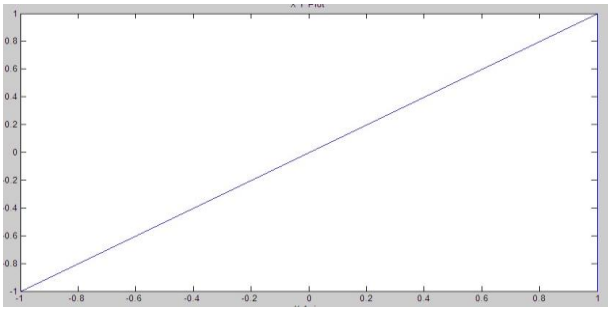

Fig.6: Synchronization Error $(\theta = 2\,)$

Fig 11: $m(t\,)$ as a function of $m'(t\,)$

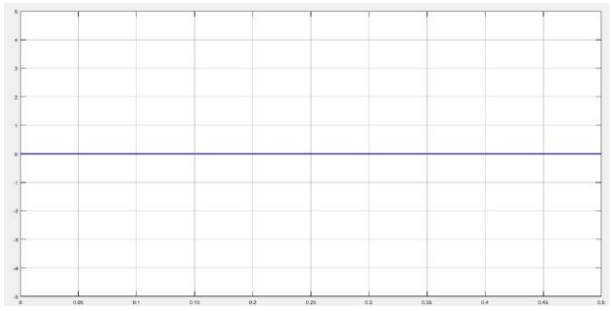

Fig 12: Synchronization error between $x_1(t)$and$x_1(t)$of the 7D system of sixorder

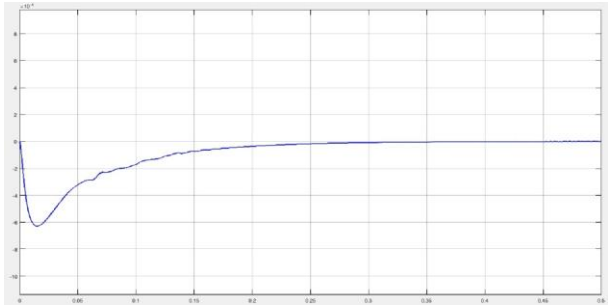

Fig 13: Synchronization error between $x_1(t)$and$\widehat{x}_1(t)$ resized of the 7D system of sixorder

From the results shown in the figures above, for our studied hyperchaotic system, it can be seen that with the high-gain observer synchronization, it is possible to reconstruct the information signal with a rather low synchronization error.

VIII. CONCLUSION

Given that the mathematical model reflects the actual dynamics of our new hyperchaotic system, this research article shows the possibility of using our system to secure the information using a high gain observer.

In this paper, a seven dimensional six order hyperchaotic system was developped, and the following conclusions are obtained through the analysis of Lyapunov exponent and the system simulations:

- The seven dimensional system is hyperchaotic, it have at least two positive lyapunovexposents.
- The synchronization was approved in favor of the errors that tend to zero when the observer gain grows.

From the results shown in the figures above, for the hyperchaotic system studied, it is noted that with the synchronization by observer with a large gain, it is possible to reconstruct the information signal with a fairly low synchronization error.

It is found that the reconstruction of the message signal is more perfect for the highorder hyperchaotic signal.

IX. REFERENCES

i. *Carroll TL,Pecora LM. Synchronization in chaotic systems.Phys Rev Lett1990;64:821–4.*

ii. *Chen G, Dong X. From chaos to order. Singapore: World Scientific; 1998.*

iii. *S.N.Lagmiri, M.Amghar, N.Sbiti, "Seven Dimensional New Hyperchaotic Systems: Dynamics and Synchronization by a High Gain Observer Design", International Journal of Control and Automation Vol. 10, No. 1 (2017), pp.251-266, Volume 10, No. 1, January 2017.*

iv. *Chen HK, Lee CI. Anti-control of chaos in rigid body motion. Chaos, Solitons& Fractals 2004;21:957–65.*

v. *S.N.Lagmiri, M.Amghar, N.Sbiti, "Synchronization between a new chaotic system andRössler system by a high gain observer", IEEEXplore, December 2014.*

vi. *K. M. Cuomo, A. V. Oppenheim, and S. H. Strogatz, IEEE Trans. Circuits Syst. 40, 626, 1993.*

vii. *L. Kocarev, K. S. Halle, K. Eckert, and L. O. Chua, Int. J. Bifurcation Chaos 2, 709, 1992.*

viii. *L. M. Pecora and T. L. Carroll, Phys. Rev. A 44, 2374, 1991.*

ix. *S.N.Lagmiri, H.ElMazoudi,N.Elalami "Control of Lotka-Volterra three species system via a high gain observer design", International Journal of Computer Applications (0975 8887) Volume 77 - No. 15, September 2013.*

x. *A. V. Oppenheim, K. M. Cuomo, and S. H. Strogatz, "Synchronization of Lorenz-based chaotic circuits with applications to communications", IEEE Trans. on CAS, part II, vol. 40, no. 10, pp. 626-633, 1993.*

xi. *Femat R. and Solis-Perales G., "On the chaos synchronization phenomena," Phys. Lett. A, 262 1999.*

xii. *S.N.Lagmiri, M.Amghar, N.Sbiti, "Hyperchaos based Cryptography: New Seven Dimensional Systems to Secure CommunicationsCirculation in Computer Science Vol.2, No.2, pp: (20-30), March 2017.*

xiii. *S.N. Lagmiri, H.ElMazoudi, N. Elalami, "Synchronization of 4-d hyperchaotic qi system by high gain observer", International Conference on Structural Nonlinear Dynamics and Diagnosis (CSNDD'2014), May 19-21, 2014. Agadir, Morocco.*

xiv. *S.N.Lagmiri, M.Amghar, N.Sbiti, "Synchronization between a new chaotic system andRössler system by a high gain observer", 14th Mediterranean Microwave Symposium December 12-14, 2014, Marrakech, Morocco.*

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