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Numerical Solution to Natural Convection in Triangular Enclosures and Its Application for Double Dome Solar Water Distillation Systems

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Abstract: ***Natural convection flow is numerically estimated in triangular domain using finite difference method. Natural convection flow in triangular domain enclosures is a common phenomenon observed in a variety of applications such as double dome type solar water distillation units, building roof top cross-section (attic space), solar collectors and in many other engineering applications. Convection flow is analyzed by varying the Rayleigh number (Ra) from 10^3 to 10^6 and the aspect ratio from 0.2 to 1.0 with bottom surface heated and inclined surfaces cooled simultaneously. The fluid inside the domain is air having Prandtl no (Pr) = 0.7. The results are presented in stream line and temperature contours. Bipartisan, counter rotating triangular and symmetric stream line contours are observed within the domain for lower values of Ra. As 'Ra' value is increased the stream line contours exhibit unsymmetrical behavior with one major contour and counter rotating minor contours. The present study provides more physical insight into the natural convection in triangular domain applications and also provide a mechanism to control the heat transfer by varying aspect ratio at the design stage.***

Keywords: Natural convection, Finite Difference method, Vorticity, Streamfunction, Triangular Domain.

Nomenclature:

Ar	Aspect Ratio (H/L)	β	Thermal expansion coefficienct
Gr	Grashoff number	ε	Numerical tolerance limit
H	Height of enclosure	θ	Non dimensional temperature
L	Length of enclosure	ν	Kinematic viscosity
n	Time step	ρ	density
Pr	Prandtl number	τ	Non dimensional time
Ra	Rayleigh number	ψ	Stream function
t	time	ω	vorticity
T	Temperature	γ	Angle of inclination
$\vec{V}$	Velocity vector	***Subscripts***	
u,v	Vel. components in x & y directions	h	hot
x,y	Transverse and normal coordinates	c	cold
α	Thermal diffusivity	I,j	X and Y coordinate indices

1. Introduction:

Natural convection in triangular enclosures is a very common phenomena observed in variety of applications such as double dome type solar water distillation units, building roof top cross-section (attic space), solar collectors and in many other engineering applications. A comprehensive review of natural convection in triangular enclosures is carried out by kamiyo et al [1] and das et al [2] and Saha et al[3]. Natural convection in isosceles triangular domain is studied by many researchers due to its variety of applications. Holtzman et al [4] has carried out experimental and numerical laminar natural convection studies in isosceles triangular enclosures with a heated horizontal base and cooled upper walls. The problem is examined over aspect ratios ranging from 0.2 to 1.0 and Grashof number (Gr) from 10^3 to 10^5.

Pitchfork bifurcation is observed at critical Gr above which the symmetric solutions are unstable to finite perturbations and asymmetric solutions are instead obtained. Basak et al. [5] used finite element method to simulate the natural convection in isosceles triangular enclosures due to uniform and non-uniform heating at the side walls. The numerical solution of the problem is presented for various Rayleigh numbers (Ra), (103 < Ra < 106) and Prandtl numbers (Pr), (0:026 < Pr < 1000). It has been found that at small Prandtl numbers, geometry does not have much influence on flow structure while at Pr = 1000, the stream function contours are nearly triangular showing that geometry has considerable effect on the flow pattern. In addition, the presence of multiple circulations are observed for small Pr = 0.026 which causes wavy distribution of local Nusselt number. It is observed that non-uniform heating produces greater heat transfer rates at the center of the walls than the uniform heating; however, average Nusselt numbers show overall lower heat transfer rates for the non-uniform heating case. Taher et al[6] used Lattice Boltzamann method (LBM) for simulating similar problem and studied the effect of varying the Ra and Aspect ratio. Saha et al [7] has studied the Natural convection in a triangular enclosure heated from below and non-uniformly cooled from top. The numerical simulations of the unsteady flows over a range of Rayleigh numbers and aspect ratios are carried out using Finite Volume Method. Since the upper inclined surfaces are linearly cooled and the bottom surface is heated, the flow is potentially unstable. It is revealed from the numerical simulations that the transient flow development in the enclosure can be classified into three distinct stages; an early stage, a transitional stage, and a steady stage. The flow inside the enclosure depends significantly on the governing parameters, Rayleigh number and aspect ratio. The effect of Rayleigh number and aspect ratio on the flow development and heat transfer rate are discussed. The key finding for this study is to analyze the pitchfork bifurcation of the flow about the geometric center line. The overall studies are found to be in good agreement and have been able to consistently predict the natural convection flow in triangular domain.

Similar studies were conducted for non-isosceles and inclined triangular domains. Mahmoudi et al. [8] conducted Numerical Study of Natural Convection in an right-angled triangular enclosures for Different Thermal Boundary Conditions using Lattice Boltzmann method. Numerical results are obtained for a wide range of parameters: the Rayleigh number spanning the range (10^3 - 10^6) and the inclination angle varying in the intervals (0° to 120°) and (0° to 360°) for two cases adiabatic vertical walls and inclined isothermal walls. It is observed that inclination angle can be used as a relevant parameter to control heat transfer in right-angled triangular enclosures.

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Solar distillation in double dome and single dome structures are well studied using double diffusive convection in triangular enclosures. Omri et al [9] has studied the Natural convection effects in solar stills. The aim of the study is to examine the thermal exchange by natural convection and effects of buoyancy forces on flow structure. The study provides useful informations on the flow structure sensitivity to the governing parameters, the Rayleigh number and the tilt angle, on the thermal exchange. In a basin still receiving a uniform heat flux, the results show that the bottom is not isotherm and the flow structure is sensitive to the cover tilt angle. Many recirculation zones can occur in the core of the cavity and the heat transfer is dependent on the flow structure. The results of this study can provide information for the enhancement of the design of the energy systems such as solar water distillers and air conditioning systems. Rahman et al [10] has studied the Double-diffusive natural convection in a triangular solar collector. Effects of the thermal Rayleigh number and buoyancy ratio are presented by streamlines, isotherms, isoconcentration as well as local and mean heat and mass transfer rates for the aforesaid parameters. Effects of the thermal Rayleigh number and buoyancy ratio are presented by streamlines, isotherms, isoconcentration as well as local and mean heat and mass transfer rates for the aforesaid parameters.

Varol et al. [11] has conducted study on Natural convection in triangular enclosures with protruding isothermal heater. Governing parameters, which are effective on flow field and temperature distribution, are; Rayleigh number, aspect ratio of triangle enclosure, dimensionless height of heater, dimensionless location of heater and dimensionless width of heater. Streamlines, isotherms, velocity profiles, local and mean Nusselt numbers are presented. It is found that all parameters related with geometrical dimensions of the heater are effective on temperature distribution, flow field and heat transfer.

In the present study, Convection flow is analyzed by varying the Rayleigh number (Ra) from 10^3 to 10^6 and the aspect ratio (base length/height) from 0.2 to 1.0 with bottom surface heated and inclined surfaces cooled simultaneously. The fluid inside the domain is air having Prandtl no (Pr) = 0.7. The results are presented in stream line and temperature contours.

2. Problem Formulation

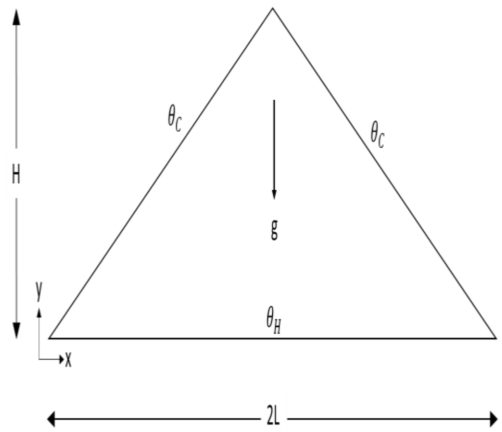

Figure1: The triangular domain model

Consider a 2D triangular domain of base length ‘2l’ and height ‘H’ as shown in Fig. 1. The cavity is filled with air and its bottom and inclined walls are maintained at ‘θ_h’ and ‘θ_c’, respectively. Boundary conditions for a triangular enclosure is shown in figure [1].

A 2D Laminar Natural Convection flow is assumed inside the cavity. Boussinesq approximation is assumed for the gravity term in the momentum equation. The flow is governed by the following set of equations.

Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Conservation of X-directional Momentum with Boussinesq approximation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - g\beta_T(T - T_C)sin\gamma$$

Conservation of Y-directional Momentum with Boussinesq approximation

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta_T(T - T_C)cos\gamma$$

Conservation of Energy

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_c\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

Average Nusselt Number

$$\overline{Nu} = -\frac{1}{L}\int_0^L \frac{\partial T}{\partial x} dy$$

Grashoff Number: Gr = Ra X Pr

Boundary conditions for a triangular domain are as follows

$x = x, y = 0: u = 0, v = 0, T = T_h$
$x = x, y = x: u = 0, v = 0, T = T_c \ \forall\ (x \le l)$
$x = x, y = (2l - x), u = 0, v = 0, T = T_c \ \forall(x > l)$

2. Numerical Method

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The governing equations are solved in a 2D triangular domain. Vorticity-Stream function formulation is used for solving the Governing equations. Obtained Partial differential equations for Stream function, vorticity and Temperature are converted to Algebraic equations using Finite difference method. Staircase approximation is used to solve the finite difference method applied to non-rectangular geometry. 1st order Upwind scheme is used for discretization of convective terms. 2nd order Central difference scheme is used for discretization of diffusion terms. 2nd Order Alternate Direction Implicit (ADI) scheme is used to discretize the transient term. The obtained coefficient matrices are in implicit line Tri-diagonal form and are solved using Thomas algorithm. The numerical computations are carried out for 154X77 grid nodal points for a time step of 10-4. The convergence criteria required that the absolute difference between the current and previous iterations for all of the dependent variable be less than 10-5. Grid Independence test is carried out and found 154X77 grid density gives satisfactory performance for the present study. The average Nusselt number is calculated at the bottom surface.

4.Results

The results obtained for triangular domain are compared with Holtzman et al.[4]. for the Average Convective Nusselt number parameter. This parameter is defined as given below. Table [1] shows the comparision of $\overline{Nu_c}$ for various Aspect ratios in the range of 0.2 to 1.0. and for Grashoff Number in the range of 10^3 to 10^5 . Figure [2] shows the comparision of local convective Nusselt number (Nu_c) along the Symmetric plane of Isosceles triangular domain for Ar = 0.5, Gr = 10^5

Average Convective Nusselt Number $\overline{Nu_c}$:

$$\overline{Nu_c} = \frac{1}{2A_r}\int_0^{2A_r} Nu_c(X)dX$$

where Convective Nusselt Number is the ratio of Nusselt number at given Gr to Nusselt number evaluated for corresponding conduction solution I.e Gr = 0 given by

$$\overline{Nu_c} = \frac{Nu_c|_{Gr}}{Nu_c|_{Gr=0}} \;;\; Nu_c|_{Gr} = \left[-\frac{\partial\theta}{\partial Y}\right]_{Y=0} \text{ and } A_r = \frac{H}{L} \text{ for}$$

triangular domain shown in figure []

Grid Independence study:

The governing equations are solved in a 2D triangular domain. The numerical computations are carried out for 154X77 grid nodal points for a time step of 10-4. The convergence criteria required that the absolute difference between the current and previous iterations for all of the dependent variable be less than 10-5. Grid Independence test is carried out for various grid sizes from 101X51, 154X77, 201X101 and found that 154X77 grid density gives satisfactory performance for the present study.

Table1 : Comparision of Average Convective Nusselt Number, with holtzman et al.:

	Aspect Ratio	Gr = 10^3	Gr = 10^4	Gr = 10^5
Holtzman et al.	1.0	1.0	1.07	1.80
Present Study		0.996	1.08	1.85
Holtzman et al.	0.5	1.0	1.20	2.19
Present Study		0.99	1.20	2.20
Holtzman et al.	0.2	1.0	1.28	2.48
Present Study		0.998	1.29	2.45

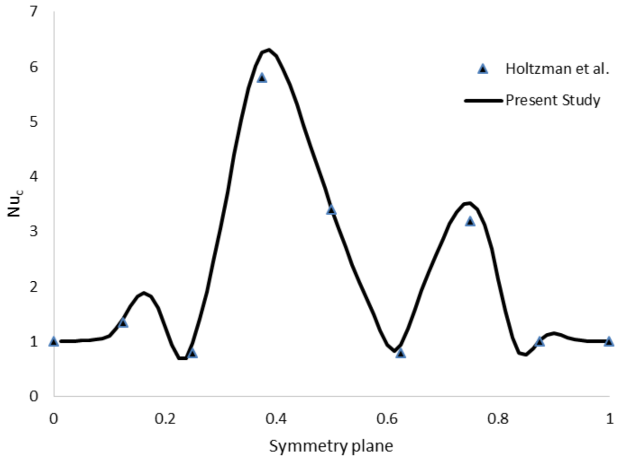

Figure 2 : Comparision of local convective Nusselt number with Holtzman et al.

The non-dimensional streamline and temperature contours are shown in the following figures [3 - 10] by varying the Rayleigh number from Ra = 10^3 to 10^5 . The effect of changing the angle of inclination of inclined walls is observed in figure [9-10] for Ra = 10^6 and inclination angle of inclined walls = 30°.

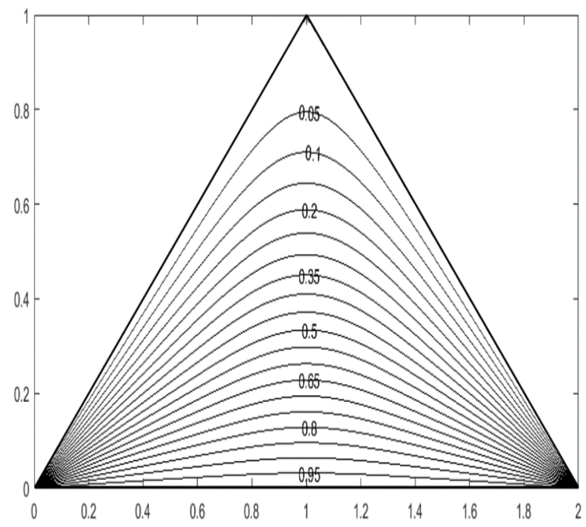

Figure 3: Natural convection flow Isotherms with Aspect ratio:1.0, Ra = 10^3

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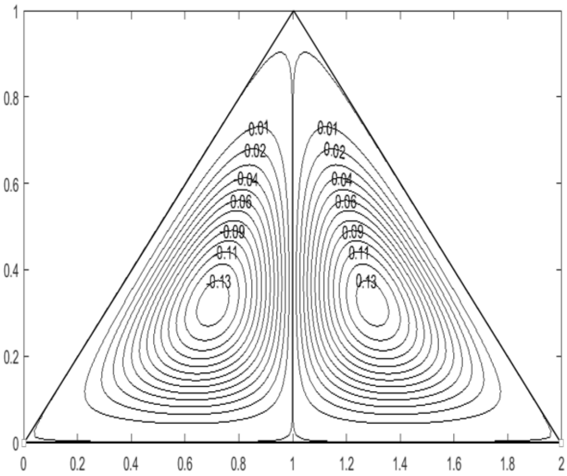

Figure 4: Natural convection flow Streamlines with Aspect ratio:1.0, Ra = 10^3

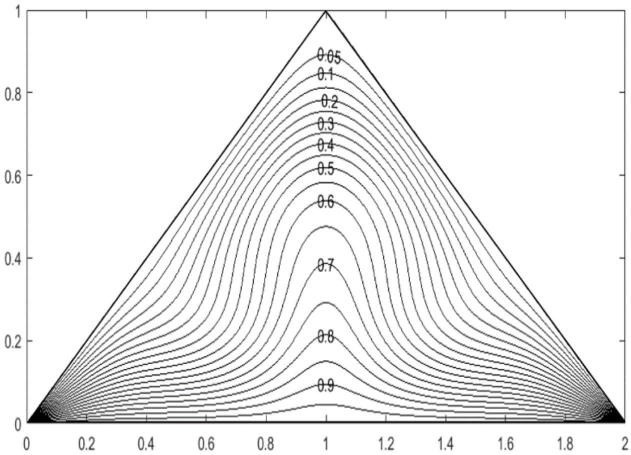

Figure 5: Natural convection flow Isotherms with Aspect ratio:1.0, Ra = 10^4

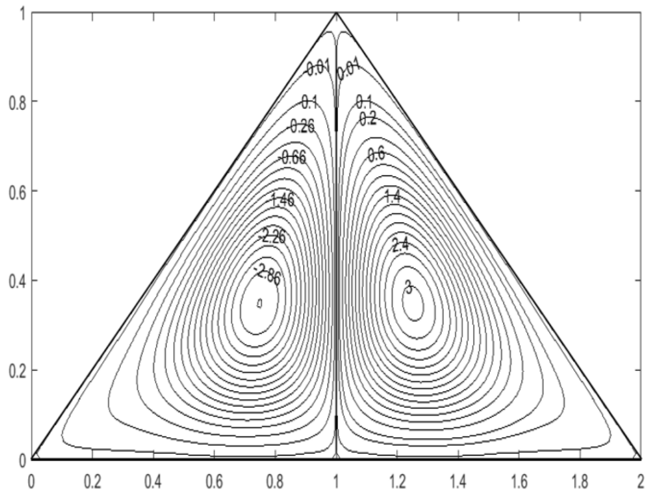

Figure 6: Natural convection flow Streamlines with Aspect ratio:1.0, Ra = 10^4

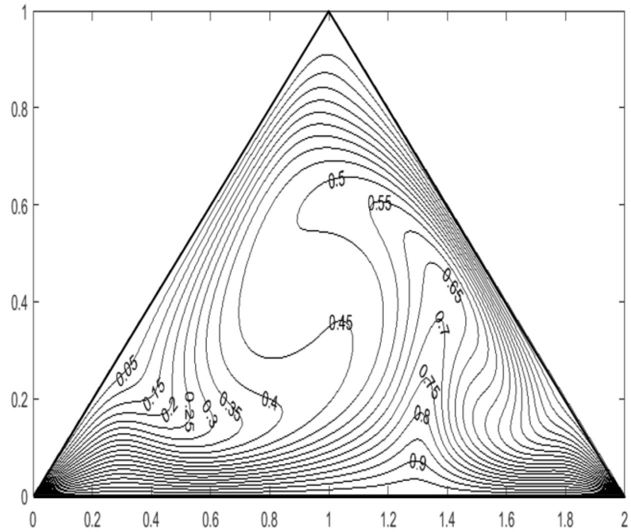

Figure 7: Natural convection flow Isotherms with Aspect ratio:1.0, Ra = 10^5

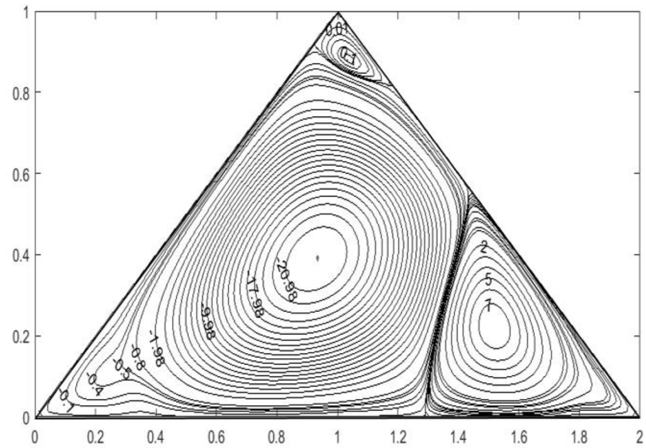

Figure 8: Natural convection flow Streamlines with Aspect ratio:1.0, Ra = 10^5

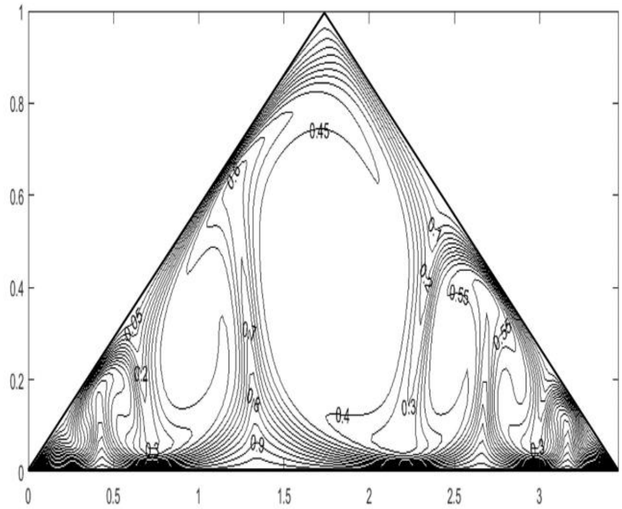

Figure 9: Isotherms for Ra = 10^6 and angle of inclination = 30°.

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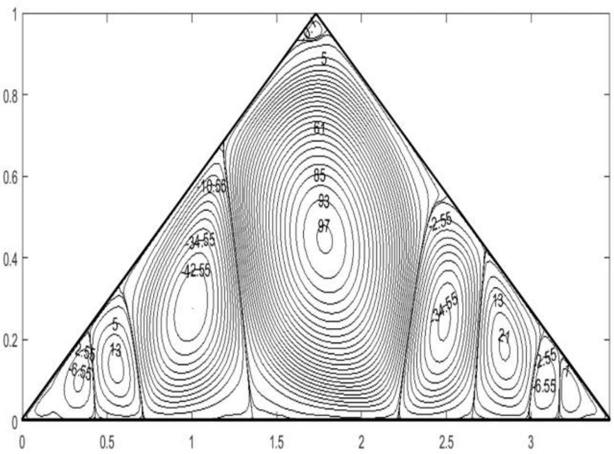

Figure 10: Streamline and Isotherms for Ra = 10^6 and angle of inclination = 30°.

Conclusions:

This paper presents a method for solving viscous incompressible Navier-stokes equations in vorticity stream-function formulation and its application to natural convection flow in triangular domain.

Convection flow is analyzed by varying the Rayleigh number (Ra) from 10^3 to 10^6 and the aspect ratio from 0.2 to 1.0 with bottom surface heated and inclined surfaces cooled simultaneously. Such configuration is commonly encountered in solar water distillation systems of double dome type.

The results are presented in stream line and temperature contours. At low temperature difference between bottom wall and inclined walls the heat raises at the symmetric plane at center of triangular domain and cools near the inclined walls and thus completing a rotating contours which are symmetric in nature. Bipartisan, counter rotating triangular and symmetric stream line contours are observed within the domain for lower values of Ra. As ‘Ra’ value is increased the stream line contours exhibit unsymmetrical behavior with one major contour and counter rotating minor contours.

The developed method estimates the Natural convection flow behaviour in triangular domain. The present study can be applied for optimizing the design of Solar water distillation systems.

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