

Performance of Energy Detection based Spectrum Sensing using Diversity Techniques over Rayleigh Fading Channel

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Abstract- Energy Detection based Spectrum Sensing technique plays major role because of its low complexity. Performance of Energy detection based spectrum sensing of unknown signals can be achieved over Rayleigh fading channel by deriving its detection probability with and without considering diversity techniques. Detection probabilities versus average Signal-to-Noise Ratio (SNR) plots are obtained for Rayleigh, Nakagami-m and Rician fading channels for the no-diversity case. Similar plots for Rayleigh channel are plotted using various diversity techniques such as Equal Gain Combining (EGC), Switch and Stay Combining (SSC) and Selection Combining (SC). Both the above cases are compared. The results show that EGC technique has better performance compared to other diversity techniques as its detection probability is high, at low SNR values (10dB). For SNR = 10dB, the difference in P_d for EGC and SSC is approximately 0.25 (0.92-0.67) and that for SSC and SC is 0.3 (0.67-0.37) and it is 0.22 (0.37-0.15) for SC and No-diversity.

Keywords- Cognitive radio, Spectrum Sensing, Energy Detection, Detection Probability, average Signal to noise ratio.

I. INTRODUCTION

Traditional wireless networks use fixed spectrum allocation policies for licensed users. Recent studies of the spectrum show that the average utilization of the spectrum is low. And this underutilization is due to the fact that a licensed user may not fully utilize the spectrum at all times in all locations. Hence to meet the increasing spectrum demands for wireless applications, flexible spectrum management techniques are needed to improve efficiency of spectrum usage. Dynamic Spectrum Access (DSA) is proposed to solve these current inefficiency problems and hence Cognitive Radio (CR) is the key enabling technology which will enable the secondary user to determine which portion of the spectrum is available and detect the presence of licensed users. This process is called Spectrum Sensing. CRs detect the unused spectrum and share it without harmful interference with the primary users. To

sense the existence of licensed user, various spectrum sensing techniques are used.

This paper focuses on Energy detection based Spectrum Sensing Technique over different Wireless Fading Channels like Rayleigh, Nakagami and Rician. Closed form expressions for probability of detection over these Fading channels are evaluated for improving the signal detection capability using receiver operating characteristics curve (ROC).

The paper is organized as follows: In Section II, the system model under consideration is described. Section III evaluates the conditional probabilities of detection and false alarm (or equivalently P_d and P_f over additive white Gaussian noise (AWGN) channels). While Section IV deduces the detection probability over various fading channels, section V studies the impact of diversity on the detection probability. Finally, simulation results and concluding remarks are offered in Section VI.

II. SYSTEM MODEL

Energy detection is the most popular spectrum sensing method since it is simple to implement and does not require any prior information about the primary signal [10]. An energy detector (ED) simply treats the primary signal as noise and decides on the presence or absence of the primary signal based on the energy of the observed signal. Since it does not need any a priori knowledge of the primary signal, the ED is robust to the variation of the primary signal. Moreover, the ED does not involve complicated signal processing and has low complexity. In practice, energy detection is especially suitable for wideband spectrum sensing [11]. The major blocks of the Energy detector are shown in Fig. 1.

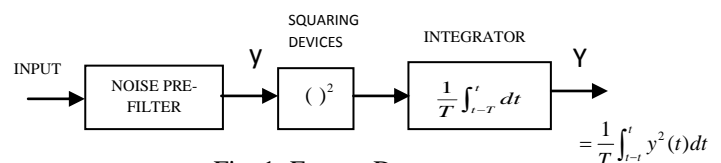


Fig. 1: Energy Detector.

The output that comes out of the integrator is energy of the filtered, received signal over the time interval and this output is considered as the test statistic to test the two hypotheses H_0

and H_1 [2]. H_0 : corresponds to the absence of the signal and presence of only noise. H_1 : corresponds to the presence of both signal and noise.

Considering the following notations: $s(t)$ is the primary signal, $r(t)$ is the received signal, $n(t)$ is the noise modeled as a zero-mean white Gaussian random process, λ is the decision threshold. The received signal $r(t)$ takes the form

$$r(t) = h s(t) + n(t) \quad (1)$$

Where $h=0$ or 1 , under hypotheses H_0 or H_1 , respectively. As described in [2], the received signal is first pre-filtered by an ideal band pass filter with transfer function given in (2) in order to limit the average noise power and normalize the noise variance.

$$H(f) = \begin{cases} \frac{2}{\sqrt{N_{01}}}, & |f - f_c| \leq W, \\ 0, & |f - f_c| > W, \end{cases} \quad (2)$$

where N_{01} is one-sided noise power spectral density, i.e., $N_{01} \equiv N_0$ in [1]. The output of this filter is then squared and integrated over a time interval to finally produce a measure of the energy of the received waveform. The output of the integrator denoted by Y will act as the test statistic to test the two hypotheses H_0 and H_1 . Although this process is of band-pass type, one can still deal with its low-pass equivalent form and eventually translate it back to its band-pass type [4]. Besides, it has been verified in [2] that both low-pass and band-pass processes are equivalent from the decision statistics perspective which is our main concern. Therefore, for convenience, we address in this paper the problem for a low-pass process. According to the sampling theorem, the noise process can be expressed as [5].

$$n(t) = \sum_{i=-\infty}^{\infty} n_i \sin c(2Wt - i), \quad (3)$$

where $\sin c(x) = \frac{\sin(\pi x)}{\pi x}$ and $n_i = n\left(\frac{i}{2W}\right)$. One can easily check that

$$n_i \sim N(0, N_{01}W), \text{ for all } i. \quad (4)$$

Over the time interval $(0, T)$, the noise energy can be approximated as

$$\int_0^T n^2(t) dt = \frac{1}{2W} \sum_{i=1}^{2u} n_i^2, \quad (5)$$

where $u=TW$ is time-bandwidth product. Where T is observation time interval (seconds) and W is one-sided bandwidth (HZ). We assume that T and W are chosen to restrict u to integer values. If we define

$$n_i = \frac{n_i}{\sqrt{N_{01}W}}, \quad (6)$$

then, the test or decision statistic Y can be written as [2].

$$Y = \sum_{i=1}^{2u} n_i^2 \quad (7)$$

Y can be viewed as the sum of the squares of $2u$ standard Gaussian variates with zero mean and unit variance. Therefore follows a central chi-square (χ^2) distribution with $2u$ degrees of freedom. The same approach is applied when the signal $s(t)$ is present with the replacement of each n_i by $n_i + s_i$ where $s_i = s\left(\frac{i}{2W}\right)$. The decision statistic Y in this case will have a non-central χ^2 distribution with $2u$ degrees of freedom and a non-centrality parameter 2γ [2]. We can describe the decision statistic as

$$Y \sim \begin{cases} \chi_{2u}^2, & H_0, \\ \chi_{2u}^2(2\gamma), & H_1, \end{cases} \quad (8)$$

The probability density function (PDF) of Y can then be written as

$$f_Y(y) = \begin{cases} \frac{1}{2^u \Gamma(u)} y^{u-1} e^{-\frac{y}{2}}, & H_0, \\ \frac{1}{2} \left(\frac{y}{2\gamma}\right)^{\frac{u-1}{2}} e^{-\frac{2\gamma+y}{2}} I_{u-1}(\sqrt{2\gamma y}), & H_1, \end{cases} \quad (9)$$

where $\Gamma(\cdot)$ is the gamma function [6, Section 8.31] and $I_V(\cdot)$ is the v th-order modified Bessel function of the first kind [6, Section 8.43].

III. DETECTION AND FALSE ALARM PROBABILITIES OVER AWGN CHANNELS

An approximate expression for P_d over AWGN channels was presented in [2]. In this section we present exact closed form expressions for both P_d and P_f . The probability of detection and false alarm can be generally computed by

$$p_d = pr(Y > \lambda | H_1) \quad (10)$$

$$p_f = pr(Y > \lambda | H_0) \quad (11)$$

where, λ is the decision threshold. We evaluated (11) using (9)

$$\text{as } p_f = \int_{\lambda}^{\infty} \frac{1}{2^u \Gamma(u)} y^{u-1} e^{-y/2} dy$$

This yields the false-alarm probability as

$$p_f = \frac{\Gamma\left(u, \frac{\lambda}{2}\right)}{\Gamma(u)}, \quad (12)$$

where $\Gamma(\cdot, \cdot)$ is the incomplete gamma function [6]. On the other hand, the probability of detection can be obtained from (9) to evaluate (10). More specifically making use of [4, Eq. (2.1-124)], the cumulative distribution function (CDF) of Y

can be evaluated (for an even number of degrees of freedom which is $2u$ in our case) as

$$F_Y(y) = 1 - Q_u(\sqrt{2\gamma}, \sqrt{y}), \quad (13)$$

Where $Q_u(a, b)$ is the generalized Marcum Q-function [7].

Hence,

$$\begin{aligned} P_d &= 1 - F_Y(y) \\ P_d &= Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \end{aligned} \quad (14)$$

IV. AVERAGE DETECTION PROBABILITY OVER FADING CHANNELS WITH NO DIVERSITY

In this section, we derive the average detection probability over Rayleigh, Nakagami, and Rician fading channels. We provide alternative expressions to those obtained in [1]. Our expressions are in closed form and are based on a different approach by averaging the conditional P_d in the AWGN case as given by (14) over the SNR fading distribution. Of course, P_f of (12) will remain the same under any fading channel since P_f is considered for the case of no signal transmission and as such is independent of SNR.

i. Rayleigh Channels

If the signal amplitude follows a Rayleigh distribution, then the SNR γ follows an exponential PDF given by

$$f(\gamma) = \frac{1}{\gamma} \exp\left(-\frac{\gamma}{\gamma}\right), \quad \gamma \geq 0, \quad (15)$$

The average P_d in this case, $\overline{P_{d \text{ Ray}}}$ can now be evaluated by averaging (14) over (15) and replacing the variable $x = \sqrt{2\gamma}$ and making use of [7, Eq. (30)] yields

$$\begin{aligned} \overline{P_{d \text{ Ray}}} &= e^{-\left(\frac{\lambda}{2}\right)} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^n + \\ &\left(\frac{1+\bar{\gamma}}{\bar{\gamma}}\right)^{u-1} \left[e^{-\frac{\lambda}{2(1+\bar{\gamma})}} - e^{-\left(\frac{\lambda}{2}\right)} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda \bar{\gamma}}{2(1+\bar{\gamma})}\right) \right] \end{aligned} \quad (16)$$

ii Nakagami Channels

If the signal amplitude follows a Nakagami distribution, then the PDF of γ follows a gamma PDF given by

$$f(\gamma) = \frac{1}{\Gamma(m)} \left(\frac{m}{\gamma}\right)^m \gamma^{m-1} \exp\left(-\frac{m}{\gamma}\right), \quad \gamma \geq 0, \quad (17)$$

where m is the Nakagami parameter. The average P_d in the case of Nakagami channels $\overline{P_{d \text{ Nak}}}$ can now be obtained by averaging (14) over (17) and then again replacing the variable $x = \sqrt{2\gamma}$ yields

$$\overline{P_{d \text{ Nak}}} = \alpha \int_0^\infty x^{2m-1} \exp\left(-\frac{mx^2}{2\gamma}\right) Q_u(x, \sqrt{\lambda}) dx \quad (18)$$

where

$$\alpha = \frac{1}{\Gamma(m) 2^{m-1}} \left(\frac{m}{\gamma}\right)^m \quad (19)$$

We can evaluate the integral in (18) by using the following relations with the aid of [7, Eq. (29)],

$$G_M = G_{M-1} + C_{M-1} F_M, \quad \text{for } \rho > -1,$$

Where

$$C_{M-1} = \frac{\Gamma\left(\frac{\rho+1}{2}\right) \left(\frac{b^2}{2}\right)^{M-1} e^{-b^2/2}}{2(M-1)! \left(\frac{p^2+a^2}{2}\right)^{\frac{\rho+1}{2}}},$$

and

$$F_M = {}_1F_1\left(\frac{\rho+1}{2}; M; \frac{b^2}{2} \frac{a^2}{p^2+a^2}\right)$$

We can evaluate G_M iteratively as follows

$$\begin{aligned} G_M &= G_{M-1} + C_{M-1} F_M \\ &= G_{M-2} + C_{M-2} F_{M-1} + C_{M-1} F_M \\ &\dots \\ &= G_1 + \sum_{n=1}^{M-1} C_n F_{n+1} \end{aligned}$$

and, then $\overline{P_{d \text{ Nak}}}$ can be written as

$$\overline{P_{d \text{ Nak}}} = \alpha \left[G_1 + \beta \sum_{n=1}^{u-1} \frac{(\lambda/2)^n}{2(n!)} {}_1F_1\left(m; n+1; \frac{\lambda}{2} \frac{\bar{\gamma}}{m+\gamma}\right) \right], \quad (20)$$

Where ${}_1F_1(.,.,.)$ is the confluent hyper geometric function [6, Section 9.2],

$$\beta = \Gamma(m) \left(\frac{2\bar{\gamma}}{m+\gamma} \right)^m e^{-\lambda/2}, \quad (21)$$

And

$$G_1 = \int_0^{\infty} x^{2m-1} \exp\left(-\frac{mx^2}{2\gamma}\right) Q(x, \sqrt{\lambda}) dx \quad (22)$$

where $Q(.,.) = Q_1(.,.)$ is the first-order Marcum Q -function.

G_1 can be evaluated for integer m with the aid of [7, Eq. (25)]

$$G_1 = \frac{2^{m-1}(m-1)!}{\left(\frac{m}{\gamma}\right)^m} \frac{\bar{\gamma}}{m+\gamma} e^{-\left(\frac{\lambda}{2m+\gamma}\right)} \left[\left(1 + \frac{m}{\gamma}\right) \left(\frac{m}{m+\gamma}\right)^{m-1} \right. \\ \left. \times L_{m-1}\left(-\frac{\lambda}{2} \frac{\bar{\gamma}}{m+\gamma}\right) + \sum_{n=0}^{m-2} \left(\frac{m}{m+\gamma}\right) L_n\left(-\frac{\lambda}{2} \frac{\bar{\gamma}}{m+\gamma}\right) \right] \quad (23)$$

where $L_n(.)$ is the Laguerre polynomial of degree n [6, 8.970].

As a byproduct, we obtain an alternative expression for $\overline{P_{d Ray}}$ when setting $m=1$ in (20) and this expression is numerically equivalent to the one obtained in (16).

iii. Rician Channel

If the signal strength follows a Rician distribution, the PDF of γ will be

$$f(\gamma) = \frac{K+1}{\gamma} \exp\left(-k - \frac{(K+1)\gamma}{\bar{\gamma}}\right) I_0\left(2\sqrt{\frac{K(K+1)\gamma}{\bar{\gamma}}}\right), \quad (24) \\ \gamma \geq 0$$

where K is the Rician factor. The average P_d in the case of a Rician channel $\overline{P_{d Ric}}$ is then obtained by averaging (14) over (24) and substituting x for $\sqrt{2\gamma}$. The resulting expression can be solved for $u=1$ using [8, Eq. (45)] to yield

$$\overline{P_{d Ric|u=1}} = Q\left(\sqrt{\frac{2K\bar{\gamma}}{K+1+\bar{\gamma}}}, \sqrt{\frac{\lambda(K+1)}{K+1+\bar{\gamma}}}\right). \quad (25)$$

For $K=0$, this expression reduces to the Rayleigh expression with $u=1$.

V. AVERAGE DETECTION PROBABILITY WITH DIVERSITY RECEPTION

In this section, we address the energy detection performance when EGC, SC, or dual SSC diversity schemes are employed.

For simplicity, we focus on the case in which the diversity paths are independent and identically distributed (IID) and are subject to Rayleigh fading.

i. Equal Gain Combining

The output SNR, γ_t , of the EGC combiner is the sum of the SNRs on all branches, i.e., $\gamma_t = \sum_{l=1}^L \gamma_l$ where L is the number of diversity branches. Adding L IID non-central χ^2 variates with $2u$ degrees of freedom and non-centrality parameter $2\gamma_l$ each results in another non-central χ^2 variate with $2Lu$ degrees of freedom and non-centrality parameter $\sum_{l=1}^L 2\gamma_l$ [4, Eq. (2.1-117)].

Hence, the P_d at the EGC output for AWGN channels can be evaluated by analogy to (14) as

$$P_{d EGC} = Q_{Lu}\left(\sqrt{2\gamma_t}, \sqrt{\lambda}\right) \quad (26)$$

The PDF of γ_t for IID Rayleigh branches is known to be given by

$$f(\gamma_t) = \frac{1}{(L-1)! \bar{\gamma}^{L-1}} \gamma_t^{L-1} \exp\left(-\frac{\gamma_t}{\bar{\gamma}}\right) \quad (27)$$

The average P_d for the EGC diversity scheme, $\overline{P_{d EGC}}$, can then be obtained by averaging (26) over (27). One can notice that the PDF in (27) is similar to that in (17) when replacing each m by L and each $\bar{\gamma}$ by $L\bar{\gamma}$. This is intuitively correct since the Nakagami parameter m can be viewed as a diversity order. Hence, $\overline{P_{d EGC}}$ is equivalent to $\overline{P_{d Nak}}$ in (20) after replacing each m by L , each $\bar{\gamma}$ by $L\bar{\gamma}$ and each u by Lu .

ii. Selection Combining

In the SC diversity scheme, the branch with maximum SNR, γ_{max} , is to be selected. The PDF of γ_{max} for IID Rayleigh branches is known to be given by

$$f_{\gamma_{max}}(\gamma) = \frac{L}{\bar{\gamma}} \left(1 - e^{-\gamma/\bar{\gamma}}\right)^{L-1} e^{-\gamma/\bar{\gamma}} \quad (28)$$

This PDF can be rewritten as

$$f_{\gamma_{\max}}(\gamma) = L \sum_{i=0}^{L-1} \frac{(-1)^i}{i+1} \binom{L-1}{i} \frac{1}{\gamma^{i/(i+1)}} e^{-\frac{\gamma}{\gamma^{i/(i+1)}}} \quad (29)$$

The PDF in (29) represents a weighted sum of exponential variates each with parameter $\frac{\gamma}{i+1}$. Hence, the average P_d for the SC diversity scheme, $\overline{p_{d_{SC}}}$ can be evaluated as

$$\overline{p_{d_{SC}}} = L \sum_{i=0}^{L-1} \frac{(-1)^i}{i+1} \binom{L-1}{i} \overline{p_{d_{Ray}} \left(\frac{\gamma}{i+1} \right)}, \quad (30)$$

Where $\overline{p_{d_{Ray}} \left(\frac{\gamma}{i+1} \right)}$ is the $\overline{p_{d_{Ray}}}$ obtained in (16) with the replacement of each γ by $\frac{\gamma}{i+1}$.

iii. Switch and Stay Combining

We address in this section the evaluation of the average P_d for the dual SSC diversity scheme [9]. The PDF of the SNR at the output of the SSC with dual IID Rayleigh branches, SSC,

$$f_{\gamma_{SSC}}(\gamma) = \begin{cases} \frac{1}{\gamma} e^{-\gamma/\bar{\gamma}} (1 - e^{-\gamma_r/\bar{\gamma}}), & \gamma < \gamma_r \\ \frac{1}{\gamma} e^{-\gamma/\bar{\gamma}} (2 - e^{-\gamma_r/\bar{\gamma}}), & \gamma \geq \gamma_r \end{cases} \quad (31)$$

where γ_r is the switching threshold. The average P_d for the dual SSC diversity scheme, $\overline{p_{d_{SSC}}}$, can be obtained by averaging (14) over the PDF in (31) yielding

$$\overline{p_{d_{SSC}}} = (1 - e^{-\gamma_r/\bar{\gamma}}) \overline{p_{d_{Ray}}} + \int_{\gamma_r}^{\infty} Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \frac{1}{\gamma} e^{-\gamma/\bar{\gamma}} d\gamma \quad (32)$$

where $\overline{p_{d_{Ray}}}$ is given in (16). Relying one more time on the change of variable $x = \sqrt{2\gamma}$ in the integral part of (32) and making use of [7, Eq. (32)], $\overline{p_{d_{SSC}}}$ can be obtained in closed form as

$$\overline{p_{d_{SSC}}} = (1 - e^{-\gamma_r/\bar{\gamma}}) \overline{p_{d_{Ray}}} + e^{-\gamma_r/\bar{\gamma}} Q_u(\sqrt{2\gamma_r}, \sqrt{\lambda}) + \left(\frac{1 + \bar{\gamma}}{\gamma} \right)^{u-1} e^{-\frac{\lambda}{2(1+\bar{\gamma})}} \times \left[1 - Q_u \left(\sqrt{2\gamma_r} \frac{1 + \bar{\gamma}}{\gamma}, \sqrt{\frac{\lambda \bar{\gamma}}{1 + \bar{\gamma}}} \right) \right] \quad (33)$$

The optimal threshold γ_r^* which maximizes the $\overline{p_{d_{SSC}}}$ can be obtained by solving $\frac{\partial \overline{p_{d_{SSC}}}}{\partial \gamma_r} = 0$ in (32) yielding

$$\gamma_r^* = \frac{1}{2} \left[Q_u^{-1}(\overline{p_{d_{Ray}}}, \sqrt{\lambda}) \right]^2, \quad (34)$$

Where $Q_u^{-1}(\dots)$ denotes the inverse u th order Marcum Q -function with respect to its first argument.

VI. RESULTS

Simulation for Rayleigh channel assumes the decision threshold, $\lambda = 30$, time-bandwidth product $u=5$, for an average SNR, $\bar{\gamma}$ in the range -8dB to 20dB. For Nakagami channel, decision threshold is taken as 77, time-bandwidth product, $u=30$, and Nakagami parameter, m is taken as 2. For the Rician channel, decision threshold is chosen as 10. And for the comparison of diversity techniques, EGC, SSC, SC and no-diversity, time-bandwidth product $u=5$ and the same decision threshold is considered.

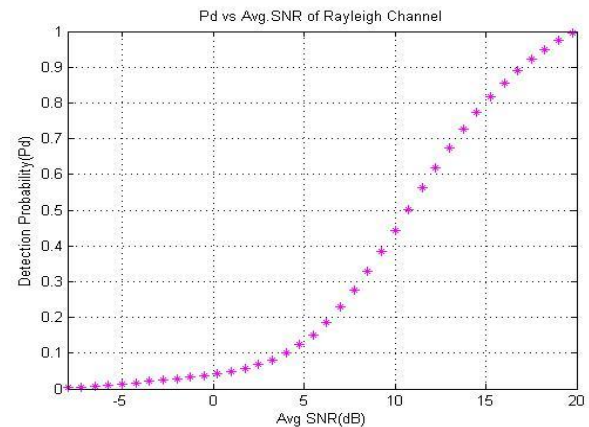


Fig 1: Probability of detection (P_d) Vs. Average SNR ($\bar{\gamma}$) for Rayleigh channel for no-diversity case.

Fig. 1 shows that the detection probability increases as average SNR ($\bar{\gamma}$) increases. P_d is the maximum for SNR = 20dB. This is the case for Rayleigh channel when no diversity technique is applied. Fig. 2 show similar plot for the Nakagami channel. The detection probability for the Nakagami channel is approximately 0.92 for an SNR as low as 8dB. For the same SNR, the Rayleigh channel has a detection probability of only 0.33. Thus, the Nakagami channel has better performance than the Rayleigh channel at low SNR values when no diversity techniques are applied.

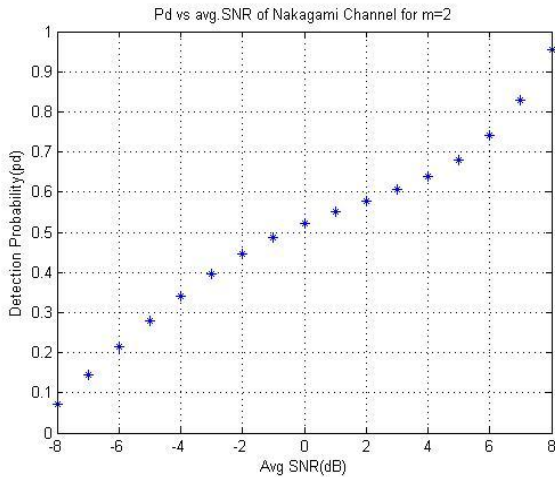


Fig 2: Probability of detection Vs. Average SNR for Nakagami channel for no-diversity case.

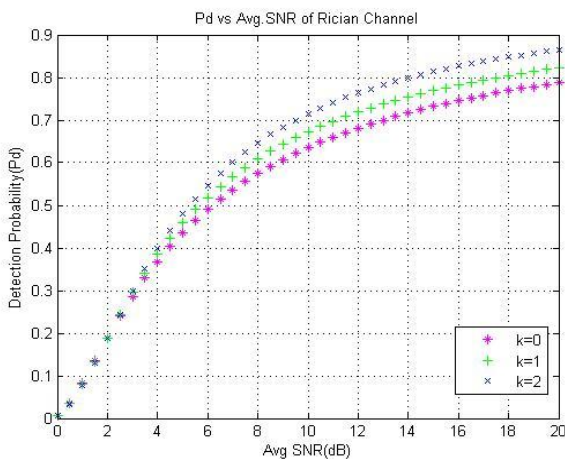


Fig. 3: Probability of detection Vs. Average SNR for Rician channel for no-diversity case.

In Fig. 3, P_d vs. $\bar{\gamma}$ is plotted for Rician channel for different values of the Rician factor, k . For a given SNR value, the detection probability increases as the Rician factor increases.

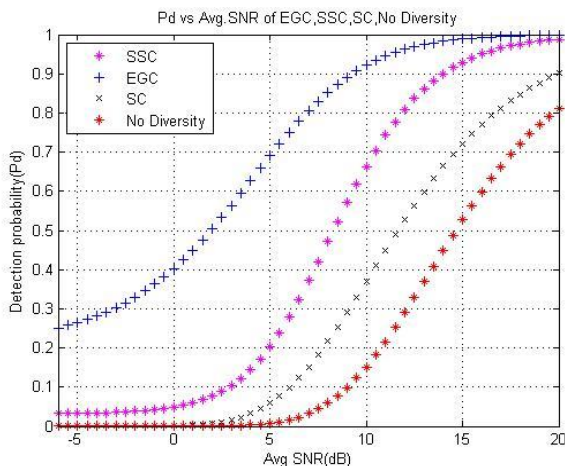


Fig. 4: Probability of detection Vs. Average SNR using various diversity techniques for Rayleigh channel including no diversity case.

From the results shown in Fig. 4, the detection probability versus average Signal to noise ratio with different diversity techniques are compared with the no-diversity case. The EGC technique possesses maximum detection probability for the given SNR, followed by SSC and SC. We get the least detection probability when no diversity technique is applied. For SNR = 10dB, the difference in p_d for EGC and SSC is approximately 0.25 (0.92-0.67) and that for SSC and SC is 0.3 (0.67-0.37) and it is 0.22 (0.37-0.15) for SC and No-diversity. For high SNR values, the Probability of detection for all the cases converges.

VII. CONCLUSION

Performance of Energy detection based spectrum sensing of unknown signals is examined over Rayleigh fading channel by deriving its detection probability with and without considering diversity techniques. Performance of Rician and Nakagami channels is verified without applying any diversity technique. The results show that, for Rayleigh channel, applying EGC technique yields better performance compared to other diversity techniques as its detection probability is high, for low SNR values. In this paper, diversity techniques are applied only to Rayleigh fading channel. The same can be extended to Rician and Nakagami channels also.

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