

Reducing Peak to Average Power Ratio of OFDM Signals using Tukey Window Technique

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Abstract- In this paper, a new and efficient technique is proposed called Tukey Window Pulse shaping technique for the reduction of PAPR in 4G OFDM system. It is based on proper selection of the time-limited waveforms of different subcarriers. Here, only one IFFT is used in the transmitter side instead of several IFFTs for each subcarrier. So this technique is simpler compared to other methods. The implementation complexity and the cost of the transmitter are significantly low. The PAPR of OFDM system using Tukey Window Technique and Sine Pulse is also compared in this paper.

Keywords- OFDM, PAPR, IFFT, Tukey Window, Sine Pulse.

I. Introduction

For achieving high data rate in 4G wireless communication system, Orthogonal Frequency Division Multiplexing (OFDM) is the most widely used technique which modulates multiple carriers. It is a parallel transmission scheme. The basic principle of OFDM is to split a high-rate serial data stream into a set of low-rate sub-streams. Even though their spectra overlap, these transmitted multiple carriers can be demodulated orthogonally. The major advantages of OFDM system are robustness to inter symbol interference, offers high spectral efficiency, multiple delay speed tolerance, immunity to frequency selective fading channel and power efficiency. OFDM has gained popularity in a number of wireless applications including WLAN standards (e.g. HIPERLAN-2, IEEE 802.11a), WiMAX standard (802.16), Wireless Metropolitan Area Networks (WMAN), Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), 3GPP-LTE, Asymmetric Digital Subscriber Line (ADSL) and power line communications [1-4].

However, one of the major problems of OFDM system is high Peak-to-Average Power Ratio (PAPR) of the transmitted signals. Here PAPR means randomly sinusoidal leads occurred during transmission of the OFDM signal. The high PAPR introduces inter-modulation distortion and undesired out-of-band radiation because of the nonlinearity

of the high power amplifier (HPA). It causes degradation of the bit error rate (BER) and high adjacent channel interference, respectively. Therefore, it is desirable to reduce the PAPR of an OFDM signal. For this reason, a number of algorithms have been developed. The simplest is the clipping and filtering [5], [6]. However, this nonlinear process deteriorates the system performance [7]. The other common schemes are block coding [8], tone reservation and injection [9], [10], nonlinear companding transform schemes [11], interleaving [12], a partial transmit sequence (PTS) scheme [13], and selected mapping (SLM) schemes [14]. The main drawback of these schemes is that it introduces additional complexity where extra FFTs are needed and side information (SI) has to be transmitted from the transmitter to the receiver to recover the original data.

In this paper a new technique known as Tukey Window pulse shaping technique is proposed for the reduction of PAPR in OFDM system. It is based on the proper selection of the time-limited waveforms of the different subcarriers. Thus only one FFT/IFFT is needed for the transceiver. It significantly reduces the computational complexity of OFDM system. Furthermore, since pulse shaping introduces controlled inter-channel interference, optimum detectors can be designed without any loss in bandwidth efficiency and with very good performance in frequency selective fading channels [15].

The rest of this paper is organized as follows. The system model of OFDM transmission is described in section II. One of the major problems of OFDM system is high PAPR which is also included in this section. To overcome from this problem a new technique known as the Tukey window pulse shaping technique is also proposed in this section. In section III, simulation results are described for PAPR reduction. Finally conclusions are given in section IV.

II. System Description & Problem Formulation

OFDM is a special case of multicarrier modulation technique that divides the available bandwidth into many subcarriers. Each subcarrier is orthogonal to each other. Since the carriers are all sine/cosine wave, we know that area under one period of a sine or a cosine wave is zero. This is easily shown in Fig. 1.

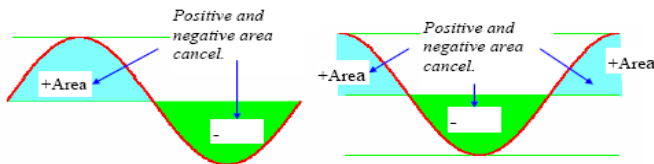


Figure 1: The area under a sine and a cosine wave over one period is always zero

The mathematical definition of orthogonality for sinusoidal harmonics is as follows:

$$\int_0^T \cos(2\pi m f_0 t) \cdot \cos(2\pi n f_0 t) dt = \begin{cases} \frac{T}{2} & (m = n) \\ 0 & (m \neq n) \end{cases}$$

$$\int_0^T \sin(2\pi m f_0 t) \cdot \sin(2\pi n f_0 t) dt = \begin{cases} \frac{T}{2} & (m = n) \\ 0 & (m \neq n) \end{cases}$$

$$\int_0^T \cos(2\pi m f_0 t) \cdot \sin(2\pi n f_0 t) dt = 0$$

where m, n are integer and the period, $T=1/f_0$, (f_0 =Fundamental frequency). For a discrete data $\{a_k\}_{k=0}^{N-1}$, the multicarrier modulated signal, $x_N(t)$, on $[0, NT]$ is represented by

$$x_N(t) = \frac{1}{N} \sum_{k=0}^{N-1} a_k e^{j2\pi f_k t} \quad (1)$$

where N is the number of subcarriers, T is the original symbol period, $\Delta f = 1/NT$ and $fk = k\Delta f$, $k = 0, \dots, N-1$. Each of these signal are individually modulated and transmitted over the channel and at the receiver the signal will be demodulated and recombine to recover the original Signal. The Fig. 2 shows the block diagram of a typical OFDM system.

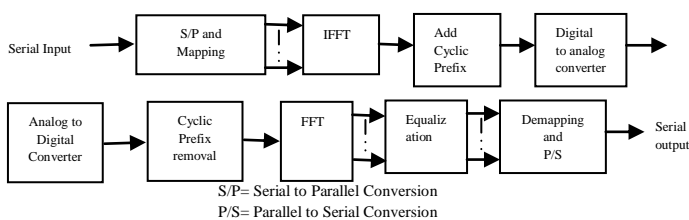


Figure 2: A typical OFDM system

A. Peak-to-Average Power Ratio of OFDM Signals

A major drawback of OFDM system is high peak-to-average power ratio (PAPR) which is defined as the maximum power to the average power of the OFDM transmission. Mathematical representation has been given below.

$$PAPR = \frac{P_{peak}}{P_{average}} = \frac{\max |x_N(t)|^2}{E[|x_N(t)|^2]} \quad (2)$$

where, P_{peak} = Peak power of the OFDM system, $P_{average}$ = average power of the OFDM system and $E[\cdot]$ is the expectation operator. Assuming uncorrelated symbols within each OFDM block, the maximum PAPR is obtained as

$$PAPR = \frac{1}{N} \max_{0 \leq t \leq T} \left(\sum_{k=0}^{N-1} |a_k(t)| \right)^2 \quad (3)$$

This is a function of the number of subcarriers N and the pulse shape used at each subcarrier. With large number of subcarriers, the maximum of the PAPR occurs with very low probability. This PAPR can be calculated in the transmitter section. The transmitter model which is used in the proposed system is shown in Fig. 3.

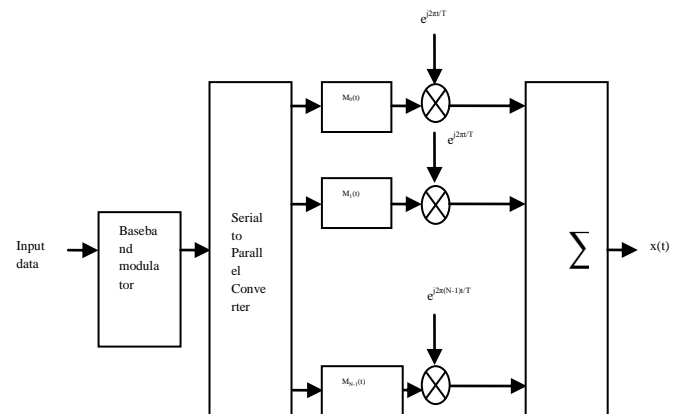


Figure 3: Transmitter of Tukey window technique in OFDM

Correlation between the different OFDM samples of the same block is a possible solution for reducing the PAPR of multicarrier OFDM signals. By making the cross correlation close to one, a multicarrier signal with very low PAPR can be obtained [16]. The cross correlation function between signal samples belonging to the same block can be defined as

$$R_m(t_1, t_2) = \frac{1}{N} \sum_{q=0}^{N-1} \sum_{r=0}^{N-1} m_{r,k}^* m_{r,k} p(t_1) p^*(t_2) e^{j2\pi 2\pi (t_1 - t_2)/T} \quad (4)$$

The cross correlation function is a function of the signal modulated symbol and the subcarrier waveforms. Hence, increasing the correlation between the OFDM signal samples of the same block can be increased through these two parameters. As a result, the PAPR of OFDM signal can

be reduced. The use of time waveforms of the different subcarriers can create an appropriate correlation which reduces the PAPR of the OFDM signal without affecting the bandwidth efficiency of the system and does not require any side information. Better reduction in PAPR of OFDM signals may be achieved by using Tukey window pulse shape.

B. Tukey window technique for PAPR Reduction

In signal processing, the **Tukey window**, also known as the **Tapered Cosine window**, is a one-parameter family of window functions used as transmit and receive windowing technique in digital communication system. It is created through the windowing of a rectangular pulse by the Tukey window. It can be regarded as a cosine lobe of width $rN/2$ that is convolved with a rectangular window of width $(1 - r/2) N$. This technique can be defined as below:

$$w[k] = \begin{cases} \frac{1}{2} [1 + \cos(\frac{2\pi}{r} (\frac{k-1}{N-1}) - \pi)] & k < \frac{r}{2}(N-1) + 1 \\ 1 & \frac{r}{2}(N-1) + 1 \leq k \leq N - \frac{r}{2}(N-1) \\ \frac{1}{2} [1 + \cos(\frac{2\pi}{r} - \frac{2\pi}{r} (\frac{k-1}{N-1}) - \pi)] & N - \frac{r}{2}(N-1) < k \end{cases} \quad (5)$$

where $k = 1$ to N

Here, r is the ratio of taper to constant sections and is between 0 and 1. If $r < 0$ then it becomes rectangular and if $r > 1$ then it becomes Hann window. The default value for r is 0.5. Different r values have been chosen to represent time domain pulse and frequency domain pulses which are shown in Fig. 4 and Fig. 5.

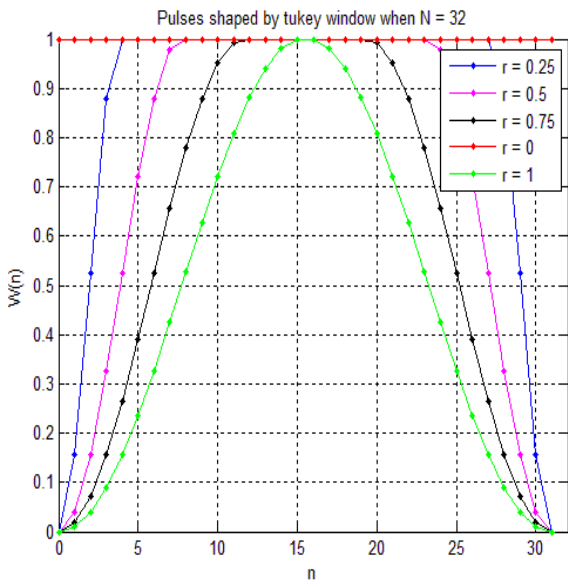


Figure 4: Time domain for varying r (Tukey Pulse)

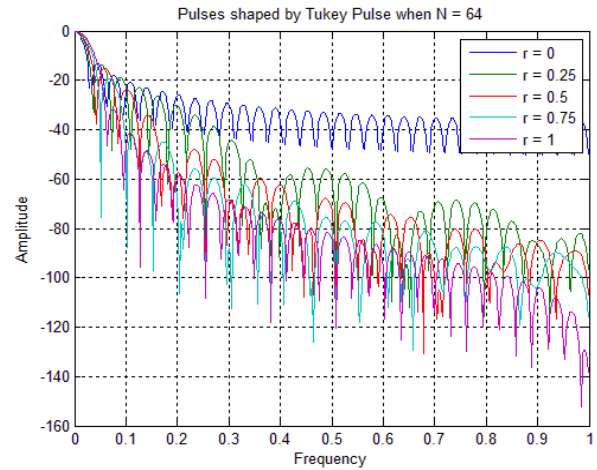


Figure 5: Frequency domain for varying r (Tukey Pulse)

The time domain and frequency domain characteristic of the Tukey window shape is examined and considered for comparisons with sine pulses [16] because Tukey window produces similar time domain shapes to the sine pulse. More specific characteristic are it narrows the time domain pulse when the parameter r is increased which is shown in the Fig. 6 and Fig. 7.

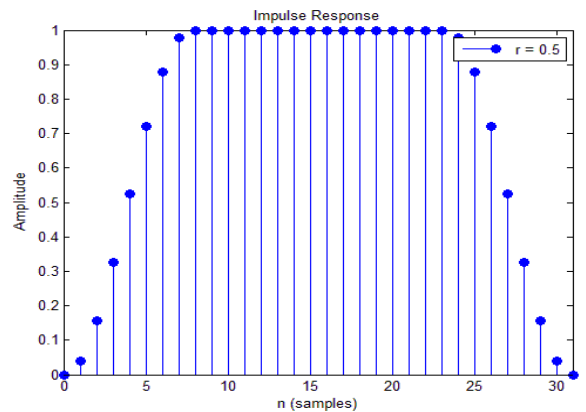


Figure 6: Impulse response when $N=32$ and $r=0.5$

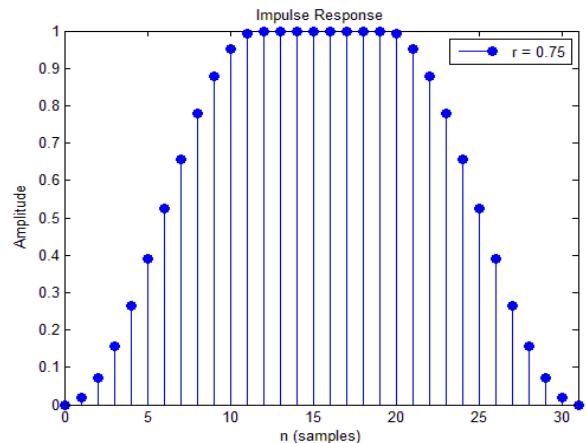


Figure 7: Impulse response when $N=32$ and $r=0.75$

III. Simulation Results

The performance of the proposed scheme is analyzed through the simulations. A very powerful and useful engineering software package is Matlab by MathWorks. It has many useful digital signal processing functions and features, which will prove to be useful in OFDM simulation. The Tukey window pulse shaping technique is implemented in OFDM system using Matlab.

The maximum PAPR of OFDM signals as a function of the parameter r is shown in Fig. 8 and Fig. 9. In Fig. 8 we can observe that the Peak to Average Power Ratio of the OFDM signal is reduced if we increase r , when the no. of subcarriers is 16 and 32. The same behavior is also observed in Fig. 9 when the no. of subcarriers is 256 and 1024. From these two figures we can also conclude that when we increase the no. of subcarrier the PAPR is also increased which coincides with the theory of high PAPR in OFDM signals.

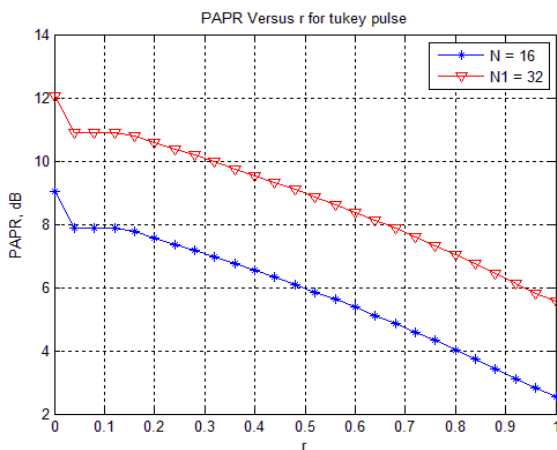


Figure 8: The maximum peak to average power ratio of OFDM signals using Tukey window technique when the number of subcarrier was 16 and 32

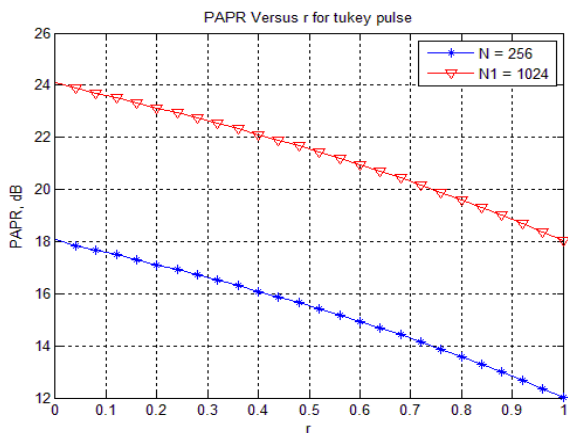


Figure 9: The maximum peak to average power ratio of OFDM signals using Tukey window technique when the number of subcarrier was 256 and 1024

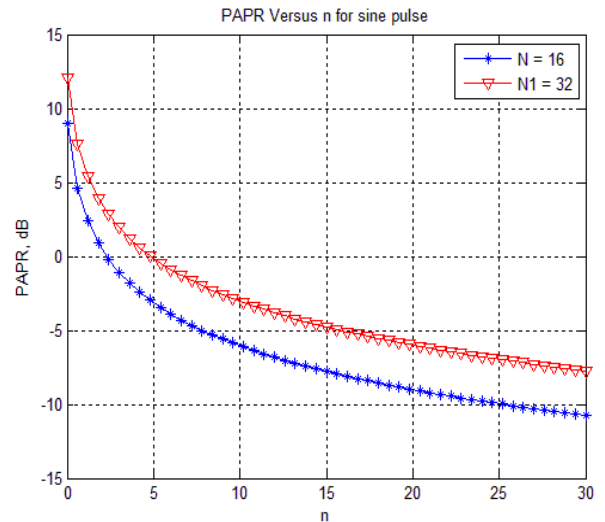


Figure 10: The maximum peak to average power ratio of OFDM signals using Sine pulse when the number of subcarrier was 16 and 32

Here we compare Tukey window pulse and Sine pulse for the reduction of PAPR in OFDM system which is shown in Fig. 8 and Fig. 10. From these figure we can conclude that by using Tukey Window pulse we get better performance. Fig. 11 illustrates the normalized cross correlation between the OFDM block samples for different values of r (0.25, 0.5, 0.75) and a total of $N = 32$ subcarriers. We notice that the correlation increases with r which is consistent with the results of Fig. 8 and Fig. 9. As mentioned earlier, higher correlation between the different samples of the OFDM block reduces the PAPR ratio of the signal.

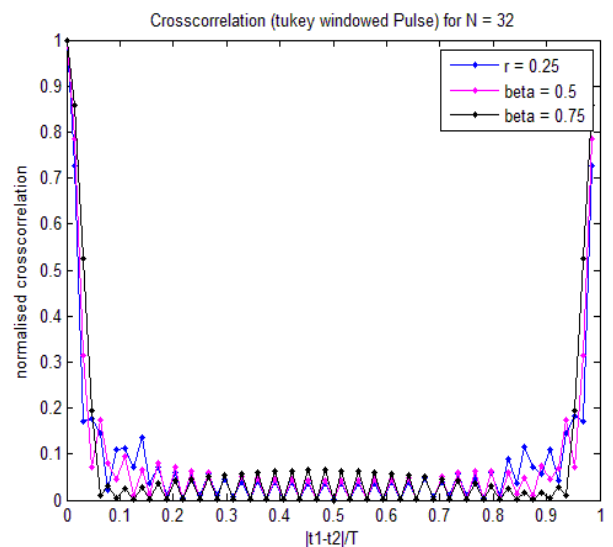


Figure 11: Normalized correlation between different OFDM samples of the same block using Tukey window technique

The results we obtain from the above simulation are suitable for the design of the time domain waveform of the OFDM system with reduced peak amplitude transmitted signal. A better PAPR is obtained by using this Tukey window pulse shaping technique.

IV. Conclusions

High PAPR of the OFDM signal causes signal distortion due to the nonlinearity of the HPA at the transmitter and affects received signal quality at the receiver. In this paper we proposed a new technique known as Tukey window technique for the reduction of PAPR in OFDM system. It is one kind pulse shaping technique which is based on proper selection of the time-limited waveforms of different subcarriers. Simulation results show that as a function of the parameter r increases, the PAPR of the OFDM signal is reduced which offer better system performance in terms of PAPR reduction of the transmitted signal. Here we also compare our proposed system with the previously proposed Sine Pulse and find better performance. The implementation complexity of this scheme is much low in compared with other scheme.

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