International Journal of Scientific Engineering and Technology (ISSN : 2277-1581)
Volume No.2, Issue No.10, pp : 991-995 1 Oct. 2013

The $Geo/G/1$ Queue Model with $\langle p, N\rangle$ Policy Set-Up Time, Multiple Vacation and Disasters

Song fang Jia, Yanheng Chen
College of Mathematics and Statistics, Chongqing Three Gorges University, Chongqing 404100, P. R. China

Corresponding Email: jiasongfang@163.com and math_yan@126.com

Abstract—***This paper studies the $Geo/G/1$ queue model with $\langle p, N\rangle$ policy set-up time, multiple vacation and disasters. The server will start up the random set-up period with the probability p if the customer number is at least N at the end of a random length V of the holiday. Otherwise, the server takes vacation repeatedly with the probability $\bar{p}$. Using embedded Markov chain and supplemental variable method, we derive the probability generating function (PGF) of the number of customers waiting in the system. From the process of the proof and the results, we also obtain the probabilities that the server is in vacations, set-up time, and busy period, respectively.***

Keywords—**Disasters, The $\langle p, N\rangle$ policy, Set-up time, Multiple vacation, $Geo/G/1$**

I. Introduction

The analysis of discrete-time queuing models has received considerable attention in the scientific literature over the past years because of its applicability widely in the real life.

Queues with disasters have been studied over decades and the work about disasters in discrete time can be found in Atencia and Moreno[1, 2], where the authors considered the single server discrete time queue with negative arrivals and various killing disciplines caused by the negative customers. And Jinting Wang and Peng Zhang [3] discussed a discrete-time retrial queue with negative customers and unreliable server. Yi and Kim etc [4] analyzed the $Geo/G/1$ queue with disasters and multiple working vacations. Park and Yang etc [5] proposed the $Geo/G/1$ queue with negative customers and disasters. Lee etc [6] studied the $Geo/G/1$ queues with disasters and general repair times in 2011. Recently, Doo and Yang [7] analyzed an $N-$policy of a discrete time $Geo/G/1$ queue with disasters and applied it to a power saving scheme in wireless sensor networks (WSNs) under unreliable network connections, where data packets are lost by external attacks or shocks.

For discrete-time queues with vacation policies, Zhang and Tian [8] treated the discrete time $Geo/G/1$ system with variant vacations in 2001. In this system, after serving all customers the server will take a random maximum number of vacations before returning to the service model. Meanwhile, the discrete-time queues exact analysis with different vacation policies can be found in Takagi [9], Tian and Alfa [10], Zhang [11], and references therein. In these papers, they all assumed that the servers stop service completely in the vacations.

The variations and extensions of these vacation models with $N-$ policy can be referred to Lee et al. [12], Krishna Reddy et al. [13], Arumuganathan, and Jeyakumar [14] introduced the optimal $N-$policy for a $M^X/G/1$ queuing system with server set-up, closedown and multiple vacations. They also proposed a cost model for a practical situation and how the results would be useful in optimizing the cost. Feinberg and Kim [15] put forward the concept of $\langle p,N\rangle$ strategy, and introduced to the $M/G/1$ queue model, among them, the $\langle p,N\rangle$ strategy research is inspired by the video stream, such as network television.
Soon afterward, X. etc [16] considered the $Geo/Geo/1$ queue model with multiple vacations, negative customers, and the $\langle p,N\rangle$ policy set-up time. However, $\langle p,N\rangle$ policy set-up time and multiple vacations queue in [16] are not involved in the model of the model $Geo/G/1$. In base of this, in this article we deal with a $Geo/G/1$ queue model with $\langle p,N\rangle$ policy set-up time, multiple vacation, and disasters：Once the system becomes empty and the customer number is less N at the end of the vacation with a random length V, the server will take vacation repeatedly with the probability $\bar{p}$. Otherwise, the server starts up the random set-up period with the probability p. And if the set-up time is in the end, the server will enter into the busy period until there is no positive customer in the system. Whenever a disaster arrives in the busy period, all present customers (i.e. the customers in the waiting line plus the customer in service) are forced to leave the system at once，and it will vanish if it arrivals to vacation period and set-up time.

The organization of the paper is as follows. In Section 2 we give model description. In Section 3 we describe the Markov process, give the one-step transition probabilities of the chain, and derive the probability generating function (PGF) of the number of customers waiting in the system by the Kolmogorov equations. We give a special case in the Section 4. From the process of the proof and the results, we also obtain the probabilities that the server is idle, set-up time, vacations, and busy period, respectively.

II. Model description

The features of the $Geo/G/1$ queue model with $\langle p,N\rangle$ policy set-up time, multiple vacation, and disasters studied here is as follows:

(1) We consider the epoch n to clarify the state of the system. Let the time axis be marked by n , $n=0,1,2,\cdots$.Then a potential customer's arrival occurs in $\left(n^-, n\right)$ and a potential customer's

departure takes place in (n, n^+), where $n^- = \lim_{\Delta n \to 0}(n - \Delta n)$ and $n^+ = \lim_{\Delta n \to 0}(n + \Delta n)$. This model is known as the late arrival model system.

(2) Two types of customers, positive and negative, arrive according to geometrical arrival processes with the parameters λ^+ and λ^-, respectively. Here we have to state the assumption regarding the order of these concurrent events because a positive customer's arrival and a negative customer's arrival can simultaneously occur at the same slot boundary. We assume that the potential negative customer's arrival occurs at $t = n^-$ before the potential positive customer's arrival. Whenever a disaster occurs, all present customers (i.e. the customers in the waiting line plus the customer in service) are forced to leave the system at once, and it will vanish if it happens in the vacation period and set-up time.

(3) There is only a server in the system, and service rule is first come first serve. The service time S of customers is independent and obeys the general discrete distribution. Its probability, PGF, and mean respectively are $s_k, k = 1,2,\mathrm{L}$, $S(z)$, and $E(S)$.

(4) One cycle begins as soon as the system becomes empty. Vacation strategy is as follows: the server will start up the random set-up period with the probability p if the customer number is less than N at the end of the holiday with a random length V. Otherwise, the server takes vacation repeatedly with the probability $\bar{p}$. And the server will enter into the busy period at the end of t he set-up time until there is no positive customer in the system. The vacation period V and the set-up time U of the server obey general distribution, and so their probabilities and PGFs are the following:

$$v_k,\ k = 1,2,\mathrm{L}\ , \qquad V(z) = \sum_{k=1}^{\infty} v_k z^k;$$

$$u_k,\ k = 1,2,\mathrm{L}\ ,\ U(z) = \sum_{k=1}^{\infty} u_k z^k\ .$$

We assume that inter arrival times, service times, set-up times, and vacation times are mutually independent. Thereinafter, for any real number $x \in [0,1]$, we denote $\bar{x} = 1 - x$.

III. Embedded Markov chain and Queue length distribution

At time $t = n^+$, the system can be described by the Markov process $\{Y_n; n \ge 1\}$ and

$$Y_n = \{C_n, \xi_{0,n}, \xi_{1,n}, \xi_{2,n}, N_n\},$$

where C_n denotes the state of the server 0, 1, and 2 according to whether the server is in vacations, set-up or busy and N_n the number of positive customers in the system. If $C_n = 0$ and $N_n \ge 0$, $\xi_{0,n}$ represents the remaining vacation time. If $C_n = 1$ and $N_n \ge N$, $\xi_{1,n}$ represents the remaining set-up time. If $C_n = 2$ and $N_n > 0$, $\xi_{2,n}$ represents the remaining service time of the customer currently being served.

It can be shown that $\{Y_n; n \ge 1\}$ is the Markov chain of our queuing system, whose state space is
$\{(0,i,k): i \ge 1, k \ge 0; (1,i,k): i \ge 1, k \ge N; (2,i,k): i \ge 1, k \ge 1;\}$.
Our object is to find the stationary distribution

$$\pi_{j,i,k} = \lim_{n \to \infty} P\{C_n = j, \xi_{j,n} = i, N_n = k\},$$

$$j = 0, i \ge 1, k \ge 0;\ j = 1, i \ge 1, k \ge N;\ j = 2, i \ge 1, k \ge 1.$$

The evolution of the chain is governed by the one-step transition probabilities

$$p_{yy'} = P\{Y_{n+1} = y' \mid Y_n = y\}.$$

If $i \ge 1, k \ge 0$, then

$$p_{(0,i+1,k)(0,i,k)} = \overline{\lambda^+},\ \ p_{(0,i+1,k)(0,i,k+1)} = \lambda^+,$$

$$p_{(0,1,k)(0,i,k)} = \overline{\lambda^+}\,\bar{p} v_i, k < N\ ,\ p_{(0,1,k-1)(0,i,k)} = \lambda^+ \bar{p} v_i,\ \ k < N\ ,$$

$$p_{(2,j,k)(0,i,0)} = \lambda^- \overline{\lambda^+} v_i,\ j \ge 1,\ p_{(2,j,k)(0,i,1)} = \lambda^- \lambda^+ v_i,\ j \ge 1,$$

$$p_{(2,1,1)(0,i,0)} = \overline{\lambda^-}\,\overline{\lambda^+} v_i\ .$$

If $i \ge 1, k \ge N$, then

$$p_{(1,i+1,k)(1,i,k)} = \overline{\lambda^+}\ ,\ p_{(1,i+1,k)(1,i,k+1)} = \lambda^+\ ,$$

$$p_{(0,1,k)(1,i,k)} = \overline{\lambda^+} p u_i\ ,\ p_{(0,1,k-1)(1,i,k)} = \lambda^+ p u_i\ .$$

If $i \ge 1, k \ge 1$, then

$$p_{(2,i+1,k)(2,i,k)} = \overline{\lambda^+}\,\overline{\lambda^-}\ ,\ p_{(2,i+1,k)(2,i,k+1)} = \lambda^+ \overline{\lambda^-}\ ,$$

$$p_{(2,1,k)(2,i,k)} = \lambda^+ \overline{\lambda^-}\ ,\ p_{(2,1,k+1)(2,i,k)} = \overline{\lambda^+}\,\overline{\lambda^-}\ ,$$

$$p_{(1,1,k)(2,i,k)} = \overline{\lambda^+}\,\overline{\lambda^-} s_i, k \ge N\ ,\ p_{(1,1,k-1)(2,i,k)} = \lambda^+ \overline{\lambda^-} s_i, k \ge N\ .$$

The Kolmogorov equations for the stationary distribution of the system are the following:

$$\pi_{0,i,0} = \pi_{0,i+1,0}\overline{\lambda^+} + \pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+} v_i + \pi_{0,1,0}\overline{\lambda^+}\,\bar{p} v_i + \lambda^- \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \pi_{2,j,k}\overline{\lambda^+} v_i,\ i \ge 1, \tag{1}$$

$$\pi_{0,i,1} = \pi_{0,i+1,0}\lambda^+ + \pi_{0,i+1,1}\overline{\lambda^+} + \pi_{0,1,0}\lambda^+ \bar{p} v_i + \pi_{0,1,1}\overline{\lambda^+}\,\bar{p} v_i + \lambda^- \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \pi_{2,j,k}\lambda^+ v_i,\ i \ge 1, \tag{2}$$

$$\pi_{0,i,n} = \pi_{0,i+1,n-1}\lambda^+ + \pi_{0,i+1,n}\overline{\lambda^+} + \pi_{0,1,n-1}\lambda^+ \bar{p} v_i + \pi_{0,1,n}\overline{\lambda^+}\,\bar{p} v_i,\ 1 < n < N, i \ge 1, \tag{3}$$

$$\pi_{0,i,n} = \pi_{0,i+1,n-1}\lambda^+ + \pi_{0,i+1,n}\overline{\lambda^+},\ n \ge N, i \ge 1, \tag{4}$$

$$\pi_{1,i,N} = \overline{\lambda^+}\pi_{1,i+1,N} + \overline{\lambda^+} p \pi_{0,1,N} u_i + \lambda^+ \pi_{0,1,N-1} p u_i,\ i \ge 1, \tag{5}$$

$$\pi_{1,i,n} = \lambda^+ \pi_{1,i+1,n-1} + \overline{\lambda^+}\pi_{1,i+1,n} + \overline{\lambda^+} p \pi_{0,1,n} u_i + \lambda^+ p \pi_{0,1,n-1} u_i,\ i \ge 1, n > N\ , \tag{6}$$

$$\pi_{2,i,1} = \lambda^+ \overline{\lambda^-}\pi_{2,1,1} s_i + \overline{\lambda^+}\,\overline{\lambda^-}\pi_{2,1,2} s_i + \overline{\lambda^+}\,\overline{\lambda^-}\pi_{2,i+1,1},\ i \ge 1, \tag{7}$$

$$\pi_{2,i,n} = \lambda^+ \overline{\lambda^-}\pi_{2,1,n} s_i + \overline{\lambda^+}\,\overline{\lambda^-}\pi_{2,1,n+1} s_i + \overline{\lambda^+}\,\overline{\lambda^-}\pi_{2,i+1,n} + \lambda^+ \overline{\lambda^-}\pi_{2,i+1,n-1},\ i \ge 1, 2 \le n < N\ , \tag{8}$$

$$\pi_{2,i,N} = \lambda^+ \overline{\lambda^-}\pi_{2,1,N} s_i + \overline{\lambda^+}\,\overline{\lambda^-}\pi_{2,1,N+1} s_i + \overline{\lambda^+}\,\overline{\lambda^-}\pi_{2,i+1,N} + \lambda^+ \overline{\lambda^-}\pi_{2,i+1,N-1} + \overline{\lambda^+}\,\overline{\lambda^-}\pi_{1,1,N} s_i,\ i \ge 1, \tag{9}$$

International Journal of Scientific Engineering and Technology (ISSN : 2277-1581)
Volume No.2, Issue No.10, pp : 991-995 1 Oct. 2013

$$\pi_{2,i,n} = \lambda^+\overline{\lambda^-}\pi_{2,1,n}s_i + \overline{\lambda^+}\overline{\lambda^-}\pi_{2,1,n+1}s_i + \overline{\lambda^+}\overline{\lambda^-}\pi_{2,i+1,n} + \lambda^+\overline{\lambda^-}\pi_{2,i+1,n-1} + \overline{\lambda^+}\overline{\lambda^-}\pi_{1,1,n}s_i + \lambda^+\overline{\lambda^-}\pi_{1,1,n-1}s_i,\ i \ge 1,\ n > N. \quad (10)$$

To solve Eqs. (1-10) above, we introduce the following generating functions:

$$\Pi_0(z,i) = \sum_{n=0}^{\infty}\pi_{0,i,n}z^n, \qquad \Pi_0^*(z,i) = \sum_{i=1}^{\infty}\Pi_0(z,i)\omega^i;$$

$$\Pi_1(z,i) = \sum_{n=N}^{\infty}\pi_{0,i,n}z^n, \qquad \Pi_1^*(z,i) = \sum_{i=1}^{\infty}\Pi_1(z,i)\omega^i;$$

$$\Pi_2(z,i) = \sum_{n=1}^{\infty}\pi_{2,i,n}z^n, \qquad \Pi_2^*(z,i) = \sum_{i=1}^{\infty}\Pi_2(z,i)\omega^i.$$

The normalizing condition is

$$\sum_{i=1}^{\infty}\sum_{n=0}^{\infty}\pi_{0,i,n} + \sum_{i=1}^{\infty}\sum_{n=N}^{\infty}\pi_{1,i,n} + \sum_{i=1}^{\infty}\sum_{n=1}^{\infty}\pi_{2,i,n} = 1.$$

Namely,

$$\Pi_0^*(1,1) + \Pi_1^*(1,1) + \Pi_2^*(1,1) = 1. \quad (11)$$

Multiplying Eqs. (1-4) by z^n and summing over n, we get that

$$\Pi_0(z,i) = z\lambda^+\Pi_0(z,i+1) + \overline{\lambda^+}\Pi_0(z,i+1) + \pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+}v_i + \lambda^-\overline{\lambda^+}\Pi_2^*(1,1)v_i + z\lambda^-\lambda^+\Pi_2^*(1,1)v_i + \overline{\lambda^+}\,\overline{p}\sum_{k=0}^{N-1}\pi_{0,1,k}z^k v_i + \lambda^+\overline{p}\sum_{k=1}^{N-1}\pi_{0,1,k-1}z^k v_i,\ i \ge 1. \quad (12)$$

Multiplying Eq. (12) by ω^i and summing over i, and taking them into Eq. (1), we can obtain

$$\frac{\omega - \overline{\lambda^+} - z\lambda^+}{\omega}\Pi_0^*(z,\omega) = -\left(\overline{\lambda^+} + z\lambda^+\right)\Pi_0(z,1) + \left[\pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+} + \lambda^-\Pi_2^*(1,1)\left(\overline{\lambda^+} + z\lambda^+\right) + \overline{\lambda^+}\,\overline{p}\sum_{k=0}^{N-1}\pi_{0,1,k}z^k + \lambda^+\overline{p}\sum_{k=1}^{N-1}\pi_{0,1,k-1}z^k\right]V(\omega) \quad (13)$$

If Eq. (13) $\omega = \overline{\lambda^+} + z\lambda^+$, then it yields

$$\Pi_0(z,1) = \frac{V\left(\overline{\lambda^+} + z\lambda^+\right)}{\overline{\lambda^+} + z\lambda^+}\left[\pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+} + \lambda^-\Pi_2^*(1,1)\left(\overline{\lambda^+} + z\lambda^+\right) + \overline{\lambda^+}\,\overline{p}\sum_{k=0}^{N-1}\pi_{0,1,k}z^k + \lambda^+\overline{p}\sum_{k=1}^{N-1}\pi_{0,1,k-1}z^k\right] \quad (14)$$

If Eq. (14) $\omega = z = 1$, we can get that

$$\Pi_0(1,1) = \left[\pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+} + \lambda^-\Pi_2^*(1,1) + \overline{\lambda^+}\,\overline{p}\sum_{k=0}^{N-1}\pi_{0,1,k} + \lambda^+\overline{p}\sum_{k=1}^{N-1}\pi_{0,1,k-1}\right] \quad (15)$$

If Eq. (14) $z = 0$, we can derive

$$\pi_{0,1,0} = \Pi_0(0,1) = \left(\pi_{2,1,1}\overline{\lambda^-} + \lambda^-\Pi_2^*(1,1) + \overline{p}\pi_{0,1,0}\right)V\left(\overline{\lambda^+}\right)$$

$$\pi_{0,1,0} = \frac{\left(\pi_{2,1,1}\overline{\lambda^-} + \lambda^-\Pi_2^*(1,1)\right)V\left(\overline{\lambda^+}\right)}{1 - \overline{p}V\left(\overline{\lambda^+}\right)} \quad (16)$$

Taking Eq. (14) into Eq. (13), it yields

$$\Pi_0^*(z,\omega) = \frac{\omega}{\omega - \overline{\lambda^+} - z\lambda^+}\left[\pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+} + \lambda^-\Pi_2^*(1,1)\left(\overline{\lambda^+} + z\lambda^+\right) + \overline{\lambda^+}\,\overline{p}\sum_{k=0}^{N-1}\pi_{0,1,k}z^k + \lambda^+\overline{p}\sum_{k=1}^{N-1}\pi_{0,1,k-1}z^k\right]\left(V(\omega) - V\left(\overline{\lambda^+} + z\lambda^+\right)\right) \quad (17)$$

If Eq. (17) $\omega = 1$, it yields

$$\Pi_0^*(z,1) = \frac{1}{(1-z)\lambda^+}\left[\pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+} + \lambda^-\Pi_2^*(1,1)\left(\overline{\lambda^+} + z\lambda^+\right) + \overline{\lambda^+}\,\overline{p}\sum_{k=0}^{N-1}\pi_{0,1,k}z^k + \lambda^+\overline{p}\sum_{k=1}^{N-1}\pi_{0,1,k-1}z^k\right]\left(1 - V\left(\overline{\lambda^+} + z\lambda^+\right)\right).$$

The generating functions above are defined for any z, $|z| \le 1$; If z =1, they can be extended by continuity

$$\Pi_0^*(1,1) = \lim_{z\to 1^-}\Pi_0^*(z,1) = E(V)\left[\pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+} + \lambda^-\Pi_2^*(1,1) + \overline{\lambda^+}\,\overline{p}\sum_{k=0}^{N-1}\pi_{0,1,k} + \lambda^+\overline{p}\sum_{k=1}^{N-1}\pi_{0,1,k-1}\right] = E(V)\Pi_0(1,1). \quad (18)$$

Applying the same method to Eqs. (5-6), we can get that

$$\Pi_1(z,i) = \left(\overline{\lambda^+} + z\lambda^+\right)\Pi_1(z,i+1) + p\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k}z^k + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}z^k\right)u_i,\ i \ge 1 \quad (19)$$

$$\frac{\omega - \overline{\lambda^+} - z\lambda^+}{\omega}\Pi_1^*(z,\omega) = -\left(\overline{\lambda^+} + z\lambda^+\right)\Pi_1(z,1) + p\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k}z^k + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}z^k\right)U(\omega) \quad (20)$$

If Eq. (20) $\omega = \overline{\lambda^+} + z\lambda^+$, it yields

$$\Pi_1(z,1) = \frac{pU\left(\overline{\lambda^+} + z\lambda^+\right)}{\overline{\lambda^+} + z\lambda^+}\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k}z^k + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}z^k\right) \quad (21)$$

If Eq. (21) $z = 1$, it yields

$$\Pi_1(1,1) = p\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k} + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}\right) \quad (22)$$

Substituting (21) into (20), it yields

$$\Pi_1^*(z,\omega) = \frac{\omega p}{\omega - \overline{\lambda^+} - z\lambda^+}\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k}z^k + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}z^k\right)\left(U(\omega) - U\left(\overline{\lambda^+} + z\lambda^+\right)\right) \quad (23)$$

If Eq. (23) $\omega = 1$, it yields

$$\Pi_1^*(z,1) = \frac{1}{(1-z)\lambda^+}\,p\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k}z^k + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}z^k\right)\left(1 - U\left(\overline{\lambda^+} + z\lambda^+\right)\right).$$

The generating functions above are defined for any z, $|z| \le 1$; When z =1, they can be extended by continuity

$$\Pi_1^*(1,1) = \lim_{z\to 1^-}\Pi_1^*(z,1) = pE(U)\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k} + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}\right)$$

International Journal of Scientific Engineering and Technology (ISSN : 2277-1581)
Volume No.2, Issue No.10, pp : 991-995 1 Oct. 2013

$$= E(U)\Pi_1(1,1) \quad (24)$$

Using the similar method to Eqs. (7-10), we can have that

$$\Pi_2(z,i) = \lambda^+\overline{\lambda^-}\Pi_2(z,1)s_i + \overline{\lambda^+}\,\overline{\lambda^-}\left(\frac{\Pi_2(z,1)}{z} - \pi_{2,1,1}\right)s_i + \overline{\lambda^+}\,\overline{\lambda^-}\Pi_2(z,i+1) + \lambda^+\overline{\lambda^-}z\Pi_2(z,i+1) + \overline{\lambda^+}\,\overline{\lambda^-}\Pi_1(z,1)s_i + \lambda^+\overline{\lambda^-}z\Pi_1(z,1)s_i,\ i \ge 1, n > N \quad (25)$$

$$\frac{\omega - \overline{\lambda^+}\,\overline{\lambda^-} - z\lambda^+\overline{\lambda^-}}{\omega}\Pi_2^*(z,\omega) = -\left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)\Pi_2(z,1) - \overline{\lambda^+}\,\overline{\lambda^-}\pi_{2,1,1}S(\omega) + \left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)\Pi_1(z,1)S(\omega) + \frac{\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}}{z}\Pi_2(z,1)S(\omega) \quad (26)$$

If Eq. (26) $\omega = \overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}$, it yields

$$\Pi_2(z,1) = \frac{zS\left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)}{\left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)\left[z - S\left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)\right]}\left[-\overline{\lambda^+}\,\overline{\lambda^-}\pi_{2,1,1} + \left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)\Pi_1(z,1)\right] \quad (27)$$

If Eq. (27) z=1, then it yields

$$\Pi_2(1,1) = \frac{S\left(\overline{\lambda^-}\right)\left[-\overline{\lambda^+}\pi_{2,1,1} + p\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k} + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}\right)\right]}{1 - S\left(\overline{\lambda^-}\right)} \quad (28)$$

If Eq. (26) $\omega = z = 1$, then it yields

$$\overline{\lambda^-}\,\overline{\lambda^+}\pi_{2,1,1} + \lambda^-\Pi_2^*(1,1) = \overline{\lambda^-}\Pi_1(1,1). \quad (29)$$

Next, we give out a lemma which is useful to prove the theorem in the sequel.

Lemma 1. The equation $z = S\left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)$ has the unique root in the range $0 < z < 1$.

Proof. For any z, $0 < z < 1$,

$$y = S\left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)$$

is an increasing function, and thus the equation

$$z = S\left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)$$

has a unique root ξ in the range $0 < z < 1$ by [7].

If the denominator of $\Pi_2(z,1)$ is zero when $z = \xi$, then the numerator should be zero under the same condition of $z = \xi$. Hence we can get that

$$\pi_{2,1,1} = \frac{pU\left(\overline{\lambda^+} + \xi\lambda^+\right)}{\overline{\lambda^+}}\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k}\xi^k + \lambda^+\sum_{k=N+1}^{\infty}\pi_{0,1,k-1}\xi^k\right). \quad (30)$$

Because of Eqs. (15) and (22), we have that

$$\overline{\lambda^+}\sum_{k=0}^{N-1}\pi_{0,1,k} + \lambda^+\sum_{k=1}^{N-1}\pi_{0,1,k-1} = \frac{\Pi_0(1,1) - \pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+} + \lambda^-\Pi_2^*(1,1)}{\overline{p}},$$

$$\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k} + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1} = \frac{\Pi_1(1,1)}{p}.$$

Since

$$\Pi_0(1,1) = \overline{\lambda^+}\sum_{k=0}^{N-1}\pi_{0,1,k} + \lambda^+\sum_{k=1}^{N-1}\pi_{0,1,k-1} + \overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k} + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}$$

$$= \frac{\Pi_1(1,1)}{p} + \frac{\Pi_0(1,1) - \pi_{2,1,1}\overline{\lambda^-}\,\overline{\lambda^+} + \lambda^-\Pi_2^*(1,1)}{\overline{p}},$$

it yields

$$\Pi_0(1,1) = \frac{\left(p\overline{\lambda^-} - \overline{p}\right)\Pi_1(1,1)}{p^2}.$$

Therefore, Eqs. (15), (22), and (29) can be converted into

$$\Pi_0^*(1,1) = E(V)\frac{\left(p\overline{\lambda^-} - \overline{p}\right)\Pi_1(1,1)}{p^2}, \quad (31)$$

$$\Pi_1^*(1,1) = E(U)\Pi_1(1,1), \quad (32)$$

$$\Pi_2^*(1,1) = \frac{\overline{\lambda^-}\Pi_1(1,1) - \overline{\lambda^-}\,\overline{\lambda^+}\pi_{2,1,1}}{\lambda^-}. \quad (33)$$

By the Eq. (11) and Eqs. (31-33), we can obtain

$$\Pi_1(1,1) = \frac{p^2\left(\lambda^- + \overline{\lambda^-}\,\overline{\lambda^+}\pi_{2,1,1}\right)}{\lambda^-\left(p\overline{\lambda^-} - \overline{p}\right)E(V) + p^2\lambda^-E(U) + p^2\overline{\lambda^-}}, \quad (34)$$

where $\pi_{2,1,1}$ satisfy with Eq. (30) .

By argument above, we can directly give out our main result in the following.

Theorem 1. Let $p(z)$ denote the PGF of the queue length distribution. Then we can obtain

$$p(z) = \Pi_0^*(z,1) + \Pi_1^*(z,1) + \Pi_2^*(z,1), \quad (35)$$

where

$$\Pi_0^*(z,1) = \frac{\left(1 - V\left(\overline{\lambda^+} + z\lambda^+\right)\right)}{(1-z)\lambda^+}\left[\overline{\lambda^-}\,\overline{\lambda^+}\pi_{2,1,1}\lambda^+(1-z) + \frac{\overline{\lambda^-}p^2\left(\lambda^- + \overline{\lambda^-}\,\overline{\lambda^+}\pi_{2,1,1}\right)\left(\overline{\lambda^+} + z\lambda^+\right)}{\lambda^-\left(p\overline{\lambda^-} - \overline{p}\right)E(V) + p^2\lambda^-E(U) + p^2\overline{\lambda^-}}z^k + \overline{\lambda^+}\,\overline{p}\sum_{k=0}^{N-1}\pi_{0,1,k}z^k + \lambda^+\overline{p}\sum_{k=1}^{N-1}\pi_{0,1,k-1}\right],$$

$$\Pi_1^*(z,1) = \frac{1}{1 - \overline{\lambda^+} - z\lambda^+}p\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k}z^k + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}z^k\right)\left(1 - U\left(\overline{\lambda^+} + z\lambda^+\right)\right),$$

$$\Pi_1^*(z,1) = \frac{\left(1 - U\left(\overline{\lambda^+} + z\lambda^+\right)\right)p}{1 - \overline{\lambda^+} - z\lambda^+}\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k}z^k + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}z^k\right) + p\overline{\lambda^-}U\left(\overline{\lambda^+} + z\lambda^+\right)\left(\overline{\lambda^+}\sum_{k=N}^{\infty}\pi_{0,1,k}z^k + \lambda^+\sum_{k=N}^{\infty}\pi_{0,1,k-1}z^k\right)\left(\frac{(1-z)S\left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)}{\left[z - S\left(\overline{\lambda^+}\,\overline{\lambda^-} + z\lambda^+\overline{\lambda^-}\right)\right]} + \overline{\lambda^-}\right),$$

International Journal of Scientific Engineering and Technology (ISSN : 2277-1581)
Volume No.2, Issue No.10, pp : 991-995 1 Oct. 2013

$$\pi_{2,1,1}=\frac{pU\left(\overline{\lambda^{+}}+\xi\lambda^{+}\right)}{\overline{\lambda^{+}}}\left(\overline{\lambda^{+}}\sum_{k=N}^{\infty}\pi_{0,1,k}\xi^{k}+\lambda^{+}\sum_{k=N+1}^{\infty}\pi_{0,1,k-1}\xi^{k}\right).$$

Corollary 1. Assume that the $Geo/G/1$, queue model with the $\langle p,N\rangle$ policy set-up time, multiple vacation and disasters is in the steady state. Then we have the following results:

(1) The probability that the server is in the vacation:

$$\Pi_0(1,1)=\overline{\lambda^{-}}p\left(\overline{\lambda^{+}}\sum_{k=N}^{\infty}\pi_{0,1,k}+\lambda^{+}\sum_{k=N}^{\infty}\pi_{0,1,k-1}\right)$$
$$+\overline{\lambda^{+}}\,\overline{p}\sum_{k=0}^{N-1}\pi_{0,1,k}+\lambda^{+}\overline{p}\sum_{k=1}^{N-1}\pi_{0,1,k-1}.$$

(2) The probability of the set-up time:

$$\Pi_1(1,1)=p\left(\overline{\lambda^{+}}\sum_{k=N}^{\infty}\pi_{0,1,k}+\lambda^{+}\sum_{k=N}^{\infty}\pi_{0,1,k-1}\right).$$

(3) The probability of the busy period:

$$\Pi_2(1,1)=\frac{S\left(\overline{\lambda^{-}}\right)\left[-\overline{\lambda^{+}}\pi_{2,1,1}+p\left(\overline{\lambda^{+}}\sum_{k=N}^{\infty}\pi_{0,1,k}+\lambda^{+}\sum_{k=N}^{\infty}\pi_{0,1,k-1}\right)\right]}{1-S\left(\overline{\lambda^{-}}\right)}.$$

IV. Special model

Case: The $Geo/G/1$ queue model with the $N-$policy and disasters.

In the $Geo/G/1$ queue model with the $\langle p,N\rangle$ policy set-up time, multiple vacation and disasters, when $p=1$ and $U=0$, V is considered as the idle period and the number of customers is not over N, and the model coverts into the $Geo/G/1$ queue model with the $N-$ policy and flushing out negative customers. The results are consistent with literature [6].

Acknowledgement

This work is supported by Youth Foundation of Chongqing Three Gorges University (Grant No.12QN-22), and the science and technology plan project of Wan Zhou district (No: 201203056).

References

i. *Atencia I, Moreno P. The discrete time $Geo/G/1$ queue with negative customers and disasters[J]. Comput Oper Res, 2004, 31(9):1537-1548.*

ii. *Atencia I, Moreno P. A single-server $G-$queue in discrete-time with geometrical arrival and service process. Performance Evaluation, 2005, 59:85–97.*

iii. *Zhu Y. J., Gu Q. F. $GM/M/1$ queue with the RCE strategy of negative customers and working vacations[J]. Journal of Jiangsu University (Natural Science Edition), 2008, 29(4): 360-364.*

iv. *X. W. Yi, J. D. Kim, D. W. Choi, and K.C. Chae, The $Geo/G/1$ queue with disasters and multiple working vacations. Stoch. Models, 2007, 23: 21–31.*

v. *H.M. Park, W.S. Yang, K.C. Chae, The $Geo/G/1$ queue with negative customers and disasters, Stoch. Models, 2009, 25: 673–688.*

vi. *D.H. Lee, W. S. Yang, H.M. Park, Geo/G/1 queues with disasters and general repair times, Appl. Math. Model, 2011, 35:1561–1570.*

vii. *D.H. Lee, W.S. Yang, The $N-$policy of a discrete time $Geo/G/1$ queue with disasters and its application to wireless sensor networks. Appl. Math. Model, 2013.*

viii. *Z.G. Zhang, N. Tian, Discrete time $Geo/G/1$ queue with multiple adaptive vacations. Queuing Syst. 2001, 38: 419–429.*

ix. *H. Takagi, Queuing Analysis: A Foundation of Performance Evaluation, Vacation and Priority Systems, Part I, vol. I, North-Holl and Amsterdam, 1991.*

x. *Alfa, A. S., Vacation models in discrete time. Queuing Systems, 2003, 44:5–30.*

xi. *Tian, N. and Zhang, Z. G.. Vacation queuing models: theory and applications. New York: Springer-Verlag, 2006.*

xii. *S.S. Lee, H.W. Lee, K.C. Chae, Batch arrival queue with N policy and single vacation, Comput. Operat. Res. 22 (1995) 173–189.*

xiii. *G.V. Krishna Reddy, R. Nadarajan, R. Arumuganathan, Analysis of a bulk queue with $N-$policy multiple vacations and setup times, Comput. Operat. Res., 1998, 25: 957–967.*

xiv. *R. Arumuganathan, S. Jeyakumar, Steady state analysis of a bulk queue with multiple vacations, setup times with N-policy and closedown times, Appl. Math. Model, 2005, 29: 972–986.*

xv. *Feinberg E A, Kim D J. Bicriterion optimization of $M/G/1$ queue with a removable server [J]. Probability in the Engineering and Informational Sciences,1996.*

xvi. *Xu Zurun, ZHU Yi-jun, Sun Guiyong, and Li Minjie , The $Geo/Geo/1$ Queue Model with Multiple Vacations, Negative Customers, and the $\langle p,N\rangle$ Policy Set-up Time[J]. Journal of engineering mathematics, 2011, 28:794-802.*