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Mathematical Programming Approach for Measuring Technical Efficiency

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Abstract: ***There has been an ever growing concern to measure efficiency of decision making units (DMUs). Parametric approaches have been the popular methods for measuring the same. Data Envelope Analysis (DEA) is an addition in this domain. This paper is an attempt to understand the concept of DEA approach. DEA is a Linear Programming Problem that provides a means of calculating apparent efficiency levels within a group of organizations. The efficiency of an organization is calculated relative to the group's observed best practice. In other words, we may say that, DEA is essentially an optimization algorithm, which develops efficiency scores for all DMUs on a scale from zero to hundred percent.***

Keywords: Production Function, Frontier, Technical efficiency, DEA, DMU.

I. Introduction

Efficiency of a firm refers to its performance in the utilization of resources at its disposal and is a relative concept. It is measured either with the desired performance of a firm or with that of any other firm. Productive efficiency of a firm combines technical efficiency and allocative efficiency. A production function presupposes technical efficiency and holds that it gives maximum possible output which can be produced from given quantities of a set of relevant inputs. A failure to produce a frontier output means that technical decision is inefficient. The amount by which a firm lies below its production frontier can be regarded as a measure of technical inefficiency.

Farrell's concept of production function can be shown in figure 1, which involves input and output values. X-axis represents the inputs while Y-axis represents outputs. If the observed input-output values are below the production frontier, it shows that firms do not attain the maximum output possible for the inputs involved for the given technology available. A measure of the technical efficiency of the firm which produces output, y, with inputs, x, denoted by point A, is given by y/y', where y' is the frontier output associated with the level of inputs, x (see point B). This is an input-specific measure of technical efficiency.

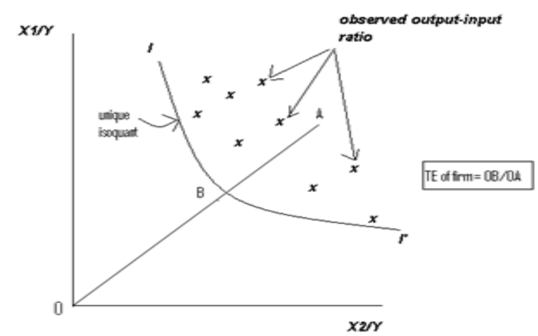

Fig 1.1: Technical Efficiency of Firms in Relative Input Space

Output-oriented technical efficiency refers to a firm's ability to obtain maximum output from a given amount of inputs. Earlier, the level of technical efficiency is measured by the distance a particular firm is from the production frontier. Thus, a firm that sits on the production frontier is said to be technically efficient. This concept is important to firms because their profits depend highly upon their value of technical efficiency. Two firms that have identical technologies and inputs but different levels of technical efficiency; will have different levels of output. The main reasons for examining technical efficiency as opposed to another type of efficiency are expressed by Kumbhakar and Lovell (2000). They state that technical efficiency is a purely physical notion that can be measured without recourse of price information and having to impose a behavioral objective on producers.

II. Data Envelope Analysis

There is an increasing concern among organizations to study level of efficiency with which they work relative to their competitors. The most common methods of comparison or performance evaluation were regression analysis and stochastic frontier analysis. These measures are often inadequate due to the multiple inputs and outputs related to different resources, activities and environmental factors. Data Envelopment Analysis (DEA) provides a means of calculating apparent efficiency levels within a group of organizations. In DEA study, efficiency of an organization is calculated relative to the group's observed best practice. DEA is essentially an optimization algorithm, which develops efficiency scores for all DMUs on a scale of zero to hundred percent, with units receiving cent percent efficiency score being called efficient.

By using linear programming approach, DEA measures the efficiency of an organization within a group relative to observed best practice within that group. The organizations can be whole agencies, separate entities within the agency or disaggregated business units within the separate entities. Before proceeding for DEA, it is important to look at the different concepts of efficiency. The most common efficiency concept is technical efficiency, that is., the conversion of physical inputs into outputs relative to best practice. In other words, given current technology, there is no wastage of inputs whatsoever in producing the given quantity of output. An organization operating at best practice is said to be 100% technically efficient. If operating below best practice levels, then the organization's technical efficiency is expressed as a percentage of best practice.

III. Data Envelope Analysis Model

To overcome the limitation of the Farrell's work, Charnes, Cooper, and Rhode (Charnes et. al., 1978) introduced their CCR DEA model that can handle multiple inputs and multiple outputs to measure technical efficiency. In the presence of multiple input and output factors, technical efficiency are defined as follows:

$$Technical\ Efficiency = \frac{weighted\ sum\ of\ outputs}{weighted\ sum\ of\ inputs}$$

Measure of the efficiency of any DMU is obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to the condition that the similar ratios for every DMU be less than or equal to unity. Assuming that there are n DMUs, each one has m inputs and s outputs, then technical efficiency of the p's DMU is given by the following model proposed by Charnes et. al. (1978):

$$\max \frac{\sum_{k=1}^{s} v_k y_{kp}}{\sum_{j=1}^{m} u_j x_{ji}}$$

$$subject\ to\ \frac{\sum_{k=1}^{s} v_k y_{kp}}{\sum_{j=1}^{m} u_j x_{ji}} \le 1 \qquad for\ all\ i$$

$$and \qquad v_k, u_j \ge 0; \qquad for\ all\ k,\ j$$

(1)

Where, $k=1$ to s, $j=1$ to m, $i=1$ to n, y_{ki} = amount of output k produced by DMU i, x_{ji} = amount of input j used by DMU i, v_k= weight assigned to output k, u_j= weight assigned to input j.

The above model is an extended nonlinear programming problem of an ordinary fractional programming problem. CCR considered the following model (2) which is the reciprocal of model (1), measures the inefficiency:

$$\min \frac{\sum_{j=1}^{m} u_j x_{ji}}{\sum_{k=1}^{s} v_k y_{kp}}$$

$$subject\ to\ \frac{\sum_{j=1}^{m} u_j x_{ji}}{\sum_{k=1}^{s} v_k y_{kp}} \ge 1 \qquad for\ all\ i$$

$$and \qquad v_k, u_j \ge 0; \qquad for\ all\ k,\ j$$

(2)

Because of the difficulty of solving fractional linear programs, Charnes et al., converted the above model into a more simplified model which is expressed below (Talluri, 2000).

$$\max \sum_{k=1}^{s} v_k y_{kp}$$
$$subject\,to \ \sum_{j=1}^{m} u_j x_{jp} = 1$$
$$\sum_{k=1}^{s} v_k y_{ki} - \sum_{j=1}^{m} u_j x_{ji} \le 0; \ for\,all\,i.$$
$$and \qquad v_k, u_j \ge 0; \qquad for\,all\,k, j$$

(3)

The previous model is executed *n* times to identify the relative efficiency scores of all DMUs involved in the evaluation. Inputs and outputs that maximize the efficiency of each DMU are selected for each DMU. The DMU is considered efficient if it obtains a score of 1, otherwise the DMU is inefficient (Cooper et. al., 2004).

In order to identify benchmarks for the inefficient DMUs, DEA provides a set corresponding efficient units that may be used as benchmarks to improve the inefficient DMUs. The solution of the following dual form of the above linear model provides the possible benchmarks for the inefficient units.

$$\min \theta$$
$$subject\,to \ \sum_{i=1}^{n} \lambda_i x_{ij} - \theta x_{jp} \le 0; \ for\,all \ j$$
$$\sum_{i=1}^{n} \lambda_i y_{ki} - y_{kp} \ge 0; \quad for\,all\,k$$
$$and \qquad \lambda_i \ge 0; \qquad for\,all\,i$$

(4)

$$where,$$
$$\theta = efficiency\,score$$
$$\lambda_s = dual\,\mathrm{var}\,iables$$

Model (3) and its dual form model (4) are known to be DEA models with constant returns to scale (CSR). CSR indicates that doubling the inputs of a DMU will result in doubling the outputs, too (SCRC, 1997). In other words there are no economies or diseconomies of scale, and that the size of the organization is not considered appropriate for measuring efficiency. To overcome this limitation of the DEA CCR model, Banker, Charnes, and Cooper extended the CCR model to handle problems with variable returns to scale (VRS). The new model, BCC, referred to by the initials of the authors, is capable of dealing with problems that exhibit decreasing, constant, and increasing returns to scale (Banker et. al.,1984).

IV: Limitations of DEA models

Some of the limitations of the DEA have been discussed by many researchers. Since DEA is a deterministic model (and descriptive in nature) it therefore provides results that are sensitive to input measurements errors. DEA attempts to measure the efficiency of a particular sample relative to best practice. Hence, it is not useful to compare the scores between two different studies. DEA results are sensitive to output and input specification, and the size of the sample. Large sample size tends to produce lower average efficiency scores. While including few DMUs relative to the number of inputs and outputs will tend to inflate the efficiency scores. Since DEA is a non-parametric approach, therefore statistical tests are not applicable.

V. Conclusion

In this paper an introduction to efficiency measurement of decision making units and the DEA methodology of measuring the same is given. With the help of a set of input and output variables from state road transport undertakings technical efficiency scores were computed both under CRS and VRS assumption along with scale efficiencies. The main advantage of DEA is that it can readily incorporate multiple inputs and outputs to calculate technical efficiency. By identifying the “peers” for organizations that are not observed to be efficient, it provides a set of potential role models that an organization can look to, in the first instance, for ways of improving its operations.

References:

(i) Banker, R. D., Charnes, A., and Cooper, A. A. (1984), Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science, 9, 1078–1092.

(ii) Charnes, A., Cooper, W.W., and Rhodes, E. (1978), Measuring the efficiency of Decision Making Units, European Journal of Operational Research, 2, 429–444.

(iii) Cooper, W., Seiford L., & Zhu, J. (2004). Data Envelopment Analysis: History, Models and Interpretations .Kluwer Academic Publishers, Boston.

(iv) Farrell, M. J. (1957).The Measurement of Productive Efficiency. Journal of Royal Statistical Society Series A 120, 253–281.

(v) Kumbhakar, S.C., C.A.K. Lovell (2000), Stochastic Frontier Analysis. Cambridge:
Cambridge University Press.

(vi) Sabah M. Al-Najjar and Mustafa A. Al-Jaybajy (2012) Application of Data Envelopment Analysis to Measure the Technical Efficiency of Oil Refineries: A Case Study, International Journal of Business Administration Vol. 3, No. 5; 2012.

(vii) Talluri, S. (2000). Data Envelopment Analysis: Models and Extensions. Decision Line, 31(3), 8-11.

(viii) Charnes A. and Cooper W.W., Programming with linear fractional functional, Naval Research Logistic Quartely, 1962, Vol. 9, No. 3-4, pp. 181-186.

(ix) Bajalinov E.B., Linear fractional programming: Methods, applications and softwares, Kluwer Academic Publisher.

(x) Sharma S.D., Operation Research, S. Chand Publication New Delhi