

# Localization using Modified Stochastic Proximity Embedding under Correlated Shadowing

<sup>1</sup>Sameera V Mohd Sagheer, <sup>2</sup>R.Nandakumar

<sup>1</sup>Department of Electronics and Communication Engineering, KMCT College of Engineering for Women, Kerala

<sup>2</sup>DOEACC, Calicut, Kerala

Emails: <sup>1</sup> sameeravm@gmail.com, <sup>2</sup> nanda@cedtic.com

**Abstract:** Localization is the process of finding the location coordinates of a node. Distances from nodes with known coordinates is required for this computation. In most of the literature, the errors in these distance measurements are assumed to be independent. However, in the real world this does not hold true. There is a need to design algorithms for the case when the errors are correlated. In this work, the Stochastic Proximity Embedding algorithm is modified to provide improved performance under such correlated errors. Simulation studies are conducted to evaluate the performance of this algorithm. Semi Definite Programming approach and the original Stochastic Proximity Embedding Algorithm are used for comparison.

**Keywords :** Stochastic, Proximity, GPS, military

## 1 Introduction

Wireless sensor networks consists of hundreds or thousands of small low power devices which can sense, compute and communicate. The network self organizes and relays information to a command post in the network. Sensor networks have a number of applications including military, disaster management, environment monitoring, and wildlife monitoring.

In any sensor network, knowledge of the locations of sensor nodes is an important requirement as the sensed data is interpreted with reference to their location. Conceptually, the simplest technique for find the locations of nodes is by equipping every node with a GPS device. However, GPS has the following drawbacks:

1. It is expensive to equip each node with GPS receivers and this increases the cost of sensors
2. Because of the delay in communication, GPS is not suitable for real time tracking

For the above reasons, it is common to equip only a small number of the nodes with GPS receivers and to find the locations of the remaining nodes with respect to these nodes. The locations of nodes with GPS receivers are assumed to be known perfectly. These are called anchor nodes. The remaining nodes with unknown location are called target nodes. The distances of all the target nodes from the nearest anchor nodes are measured. These measurements are known as ranges. The locations of the target nodes are computed using the measured ranges and the locations of the anchor nodes.

Different techniques are used for measuring ranges between anchor and target nodes - TOA (Time Of Arrival) and RSS (Received Signal Strengths). As stated above, these measurements are used for estimating location coordinates of

target nodes. However these measurements are corrupted by measurement noise. In practice this measurement noise that corrupts different link measurements are not independent. They exhibit a significant correlation between each other. Most of the localization algorithms proposed in literature do not take into account this correlation. In a few works in literature, correlation of errors is considered. In [2], the localization problem is formulated as a convex semi definite program and solved using an SDPT3 software solver.

## 1.1 Motivation: GPS

The Global Positioning System uses signals from satellites to calculate the distances to those satellites and hence compute the location of the receiver. In practice, the distance measurements are error-prone. The main sources of error are: clock bias error, ionospheric error, tropospheric error and multipath error. The ionospheric error is reported to be highly correlated in nature. An enhancement to Global Positioning System called Differential Global Positioning System (DGPS) uses a network of fixed, reference stations on the ground. The positions of these stations are known. They broadcast the difference between the fixed known positions and the positions indicated by the satellites. The receivers can then correct the distances using this information. In a wireless sensor networks the the anchor nodes are assumed to have known locations and the distances between anchor nodes can be measured. The errors in the link measurements can be found from the difference of these two. These error estimates can be used for correcting errors in other link measurements giving overall improved localization performance.

The remainder of this paper is organized as follows. Section 2 formulates the problem. Section 3 and 4 explains the correlation model and the shadowing model used. Localization using the original Stochastic Proximity Embedding Algorithm is given in Section 5. Section 6 details the proposed algorithm. The simulation results and performance comparisons are given in section 7.

## 2 Problem Formulation

A wireless network with  $m$  anchors and  $n$  sensor nodes deployed in two dimensional space is considered here. The locations of the sensor nodes are unknown and are represented as  $x_i \in R^2$  where  $i = 1, \dots, n$ . The locations of the anchor nodes are assumed to be known and are represented by  $x_k \in R^2$  where  $k = n + 1, \dots, n + m$ . The Euclidean distance between any two nodes  $i$  and  $j$  is given by

$d_{ij} = \|x_i - x_j\|$ . These distances are not known exactly. The measurements are corrupted by noise. In the scenario considered, the Received Signal Strength (RSS) gives the measure of the distance. The values of these RSS measurements are corrupted by noise.

Following is a statement of the localization problem:

Assume that we are given a set of  $p$  objects, a symmetric matrix of relationships  $d_{ij}$  between these objects, and a set of images on a 2-dimensional display plane  $(x_i, i = 1, 2, \dots, p; x_i \in R^2)$ . The problem is to place  $x_i$  onto the plane in such a way that their Euclidean distances  $d_{ij} = \|x_i - x_j\|$  approximate as closely as possible the corresponding values  $d_{ij}$ .

### 3 Log Normal Shadowing Path Loss Model

Large-scale propagation models are well suited for modeling the variations in signal strength when the separation between transmitter and receiver is large and the environment is not heavily populated, like suburban areas [4]. The attempt is to predict the long distance attenuation. This attenuation is called shadowing. Log distance path loss model and the log normal shadowing path loss model are the classical models used to model shadowing. [4]. The model used here is the log normal shadowing path loss model.

The surrounding environment clutter maybe vastly different at two different locations having the same Transmitter-Receiver separation. The log normal shadowing path-loss model is based on this fact. According to this model, for link  $i$  the received power  $P_i$  in Watts ( $W$ ) is given by [4]

$$P_i^{dB} = P_T^{dB} - PL_{d_0}^{dB} - 10\alpha \log_{10} \left( \frac{d_0}{d_i} \right) + \varepsilon_i \quad (1)$$

$P_i^{dB}$  gives the value of the received signal strength (RSS) measured. The measurement noise is accounted by the term  $\varepsilon_i$  [5]. To find the value of  $\varepsilon_i$  we the NeSh model, described in the next section is used.

The choice of the reference distance  $d_0$  is such that it is in the far-field of the transmitting antenna. But it is also chosen such that this distance is smaller than practical distances used in mobile communication systems. So  $d_0$  can be chosen from tens of kilometers in cellular networks to a few meters or even less for the case of wireless sensor networks [4]. The Frii's free-space propagation [4] is used to determine the reference path loss  $PL_{d_0}$ ,

$$PL_{d_0} = 10 \log_{10} \left( \frac{(4\pi)^2 (d_0)^2}{\lambda^2} \right) \quad (2)$$

where  $\lambda$  is the wavelength of the transmitted signal in meters.

According to the log normal shadowing model, measured signal strength at a specific transmitter-receiver distance has a Gaussian distribution about the distance dependent mean, where the signals are measured in units of  $dB$  [4]. The standard deviation of the Gaussian distribution is also in units of  $dB$ . For simplicity the links are assumed to be

symmetric. Hence the power received by node  $k$  from node  $l$  and vice versa is given by  $P_i$  where  $i$  stands for the link between node  $k$  and node  $l$  as in [2].

### 4 NeSh: Link Shadowing Correlation Model

The NeSh model [6][7][8] incorporates the correlating effects of environment. It is used to calculate the link path losses in multihop networks. Objects in the environment cause shadowing. As many links may be shadowed by same object, there is correlation between shadowing losses on geographically proximate links [5].

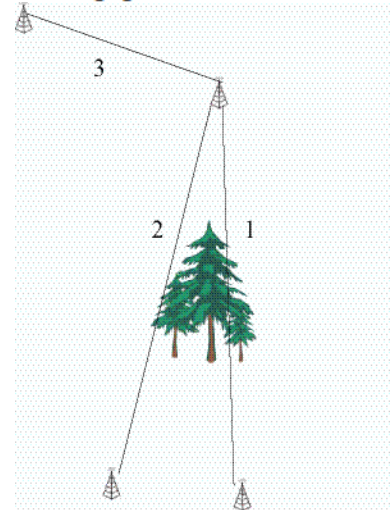


Figure 1: Example of different links experiencing similar shadowing by the same object. The shadowing experienced by links 1 and 2 is heavily correlated as the signal travels through a similar environment. Shadowing experienced by link 3, has low correlation with shadowing experienced by either 1 and 2. This is because the signal travels through a different environment.

While using the NeSh model, it is assumed that the shadowing caused by the environment is quantified in an underlying spatial loss field is an isotropic wide-sense stationary Gaussian random field with zero mean and exponentially decaying spatial correlation [6]. According to the NeSh model, covariance between two links, link  $a$  between the nodes  $S_i$  and  $S_j$  and link  $b$  between the nodes  $S_i$  and  $S_k$  can be calculated using the equation

$$\Sigma(\varepsilon_a \varepsilon_b) = \frac{\sigma^2}{\delta \sqrt{d_a d_b}} \int_{s_i}^{s_j} \int_{s_i}^{s_k} e^{-\frac{\beta - \alpha}{\delta}} d\alpha d\beta \quad (3)$$

Here  $\delta$  is a distance parameter found from measurements ( use  $\delta = 0.21m$  [6] ).  $d_a$  and  $d_b$  stand for the length of the links  $a$  and  $b$ .  $\sigma$  gives error variance.

First the covariance matrix of error is calculated with this model. The Cholesky factorization is done on this matrix. This factorized matrix is then multiplied with an uncorrelated random error. This gives the error vector with the required correlation. Let  $w$  be a vector of zero-mean, unit variance, uncorrelated random variables. The symmetric and nonnegative

definite matrix  $\Sigma$  is decomposed into  $\Sigma = LL^T$  by means of Cholesky factorization to get the correlated error vector  $\varepsilon_i = Lw$ . This correlated error is added with received signal strength measurement obtained using the log normal shadowing model [10].

### 5 WSN Localization based on Stochastic Proximity Embedding (SPE)

Stochastic Proximity Embedding (SPE) algorithm was proposed by Agrafiotis [3] to find the geometry of large and complex molecules from the distance between atoms. It is a non-linear dimensionality reduction technique. The algorithm provides a way to produce the low dimensional representation of data objects from proximity information. SPE is used in data visualization and exploratory data analysis.

Sensor network localization using SPE was proposed by A.Gopakumar and L.Jacob [1]. The algorithm requires a few anchor nodes whose locations are known accurately. The remaining are sensor nodes whose locations are unknown. The algorithm also requires knowledge of the distances between the nodes. The main steps of localization algorithm are given below:

1. Nodes collect distance information from other nodes in a two hop neighborhood
2. SPE algorithm is used on the distance matrix of Step 2, to obtain relative local map
3. The relative local map is transformed to absolute location using absolute positions of anchor nodes.

### 6 Proposed Algorithm

As stated before, shadowing experienced by different links are not independent. Links passing through same environment experience similar extent of shadowing. In the original Stochastic Proximity Embedding algorithm [1] used for localization, this correlation between different links was not considered. This causes errors in location estimates in practical scenarios. The proposed algorithm considers this effect and hence gives better results.

The algorithm is described below:

1. The lengths of the links between different anchor nodes  $(d_{S+1}, \dots, d_L)$  and the mean powers associated with the links  $(\mu_{S+1}, \dots, \mu_L)$  are calculated using the locations of anchor nodes. Given the measured link powers, the likelihood function of the remaining link powers  $\mathbf{p} = (p_1, \dots, p_L)$  and covariance matrix  $\Sigma$  is calculated as shown below

$$\mathcal{L}(\mu_1, \dots, \mu_S | \mathbf{p}, \mu_{S+1}, \dots, \mu_L, \Sigma) = \frac{1}{(2\pi)^{L/2} |\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{p} - \boldsymbol{\mu})^T \Sigma^{-1}(\mathbf{p} - \boldsymbol{\mu})\right\} \quad (4)$$

where  $L$  is the number of links,  $S$  is the total number sensor to sensor and sensor to anchor links.

The maximum likelihood estimate of  $\boldsymbol{\mu}$  denoted by  $\boldsymbol{\mu}^*$  is found by solving the following convex optimization problem [2]

$$\begin{aligned} & \text{minimize } (\mathbf{p} - \boldsymbol{\mu})^T \Sigma^{-1}(\mathbf{p} - \boldsymbol{\mu}) \\ & \text{subject to } \mu_k = P_C - \gamma \log d_k^2 \quad \forall k = S+1, \dots, L \end{aligned} \quad (5)$$

where  $P_C$  is given by

$$P_C = P_T^{dB} - PL_{d_0}^{dB} - 10\alpha \frac{\log d_0}{\log 10} \quad (6)$$

$\gamma$  is given by

$$\gamma = \frac{5\alpha}{\log 10} \quad (7)$$

and  $\mu_1, \dots, \mu_S$  are the variables of optimization. Using maximum likelihood estimates (MLE) of mean of the received link powers, MLE of the unknown link lengths  $\mathbf{d}^*$  can be computed as shown below [2]

$$d_k^* = \sqrt{\exp\left(\frac{P_C - \mu_k^*}{\gamma}\right)} \quad (8)$$

2. Now SPE algorithm is used on the distance obtained above. Here, the distances obtained in step 1 are used. The algorithm is changed as follows:

(a) Initially random coordinates  $\mathbf{x}_i$  are selected for the nodes. Learning parameter  $\lambda$  is also selected.

(b) Four distinct nodes  $i, j, k, l$  are selected at random. Let  $\mathbf{a}$  be the link between nodes  $i$  and  $j$  and  $\mathbf{b}$  be the link between nodes  $k$  and  $l$ . The distances  $\delta_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\|$  and  $\delta_{kl} = \|\mathbf{x}_k - \mathbf{x}_l\|$  are computed. If  $\delta_{ij} \neq d_{ij}$  and  $\delta_{kl} \neq d_{kl}$ , the following update is computed.

$$\mathbf{x}_i \leftarrow \mathbf{x}_i + \frac{\lambda(d_{ij} - \delta_{ij})}{2(\delta_{ij} + \epsilon)}(\mathbf{x}_i - \mathbf{x}_j)(R_2) \quad (9)$$

$$\mathbf{x}_j \leftarrow \mathbf{x}_j + \frac{\lambda(d_{ij} - \delta_{ij})}{2(\delta_{ij} + \epsilon)}(\mathbf{x}_j - \mathbf{x}_i)(R_2) \quad (10)$$

$$\mathbf{x}_k \leftarrow \mathbf{x}_k + \frac{\lambda(d_{kl} - \delta_{kl})}{2(\delta_{kl} + \epsilon)}(\mathbf{x}_k - \mathbf{x}_l)(R_1) \quad (11)$$

$$\mathbf{x}_l \leftarrow \mathbf{x}_l + \frac{\lambda(d_{kl} - \delta_{kl})}{2(\delta_{kl} + \epsilon)}(\mathbf{x}_l - \mathbf{x}_k)(R_1) \quad (12)$$

where  $\epsilon$  is a regularization parameter that makes sure division by zero does not happen. where  $R_1 =$

$$\min(\max((1 - 10\rho_{ab}(P_a - E_a)\text{sgn}(P_b - E_b)), 0.1), 1.9) \quad (13)$$

and  $R_2 =$

$$\min(\max((1 - 10\rho_{ab}(P_b - E_b)\text{sgn}(P_a - E_a)), 0.1), 1.9) \quad (14)$$

where  $\rho_{ab}$  stands for the shadowing between links  $\mathbf{a}$  and  $\mathbf{b}$  and  $E_a$  and  $E_b$  are the powers corresponding to the lengths of links  $\mathbf{a}$  and  $\mathbf{b}$  respectively.

(a) Actually, here the SPE algorithm is applied two times in one cycle considering two links and the correlation between them.

(b) Repeat Step 2S number of times. S is called

the number of steps.

(c) Decrease learning rate  $\lambda$  by  $d\lambda$ .

(d) Repeat steps 2 – 4C times. C is called the number of cycles.

1. Now the absolute locations of the sensor nodes are required. These are computed by using Procrustes algorithm [1] on anchor nodes and then by applying the transformations so obtained to the relative locations of sensor nodes

The absolute coordinates are obtained from the relative coordinates using Procrustes transformation. Procrustes transformation involves a combination of translation, rotation and/or uniform scaling. Let  $\hat{X}_{AN}$  stand for the configuration matrix of the relative coordinates of anchor nodes found using SPE. Also,  $L_{AN}$  stand for the absolute anchor node configuration. The Procrustes method attempts

to find the scaling factor  $s$ , orthogonal rotation and reflection matrix  $T$ , and the translation vector  $t$

that minimizes the loss function  $L(s, t, T) = \text{tr} [X_{AN} - (s\hat{X}_{AN}T + 1t')] [X_{AN} - (s\hat{X}_{AN}T + 1t')]$   
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subject to the condition  $T^T T = I$ , where  $\mathbf{1}$  is a vector of all  $\mathbf{1}$ 's and  $I$  is the identity matrix.

The transformations so obtained are applied to all the sensor nodes in the network to find their absolute locations.

## 7 Simulation Results and Discussion

Here, performance of proposed algorithm is evaluated using simulations and compared against Semi Definite Programming (SDP) [2]. Detailed study of the algorithm is done taking into account different factors like standard deviation of the error model, number of anchor nodes etc. The performance metric used is the localization error. This is defined as the difference between the actual location and estimated location of the sensor node.

$$\text{Localization error} = \sum_{i=N_a+1}^N \frac{\|x_i - \hat{x}_i\|}{(N - N_a)} \quad (15)$$

where  $N_a$  stands for the number of anchor nodes,  $N$  stands for the total number of nodes,  $x_i$  stands for the actual location of the node and  $\hat{x}_i$  stands for the estimated location. A network located in an area of  $1m \times 1m$  is considered here. 10 nodes are assumed. Of these, 3 are assumed to be anchor nodes. The locations of the anchor nodes are assumed to be known. Received signal strength is used to measure the inter-node distances. This received signal strength is corrupted by correlated log normal shadowing [3]. An example of the localization achieved by the proposed algorithm is shown in Fig. 2. SDP [2] gave an error of 0.7815 m while the proposed algorithm gave an error of 0.0144 m. SPE algorithm [1] gave an error of 0.0208 m. Standard deviation of 1 dB was used for the experiments.

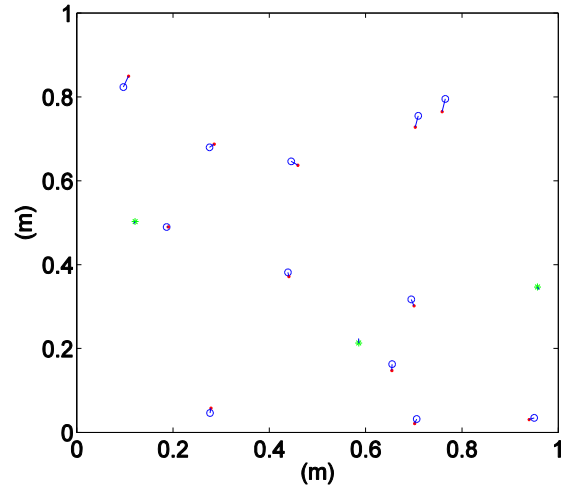


Figure 2: Output of proposed algorithm with  $\sigma = 1$ . Star stands for the location of anchor nodes, circle stands for the actual location and dot stands for the estimated location of sensor nodes. Modified SPE parameters: Learning parameter ( $\lambda$ )=1, steps=120 and cycles= 50.

### 7.0.1 Impact of Range Error

Range error is an important factor affecting localization accuracy. Impact of range error is studied. Localization errors for various values of standard error is shown in Fig. 3. The proposed algorithm is seen to give better performance than competing algorithms.

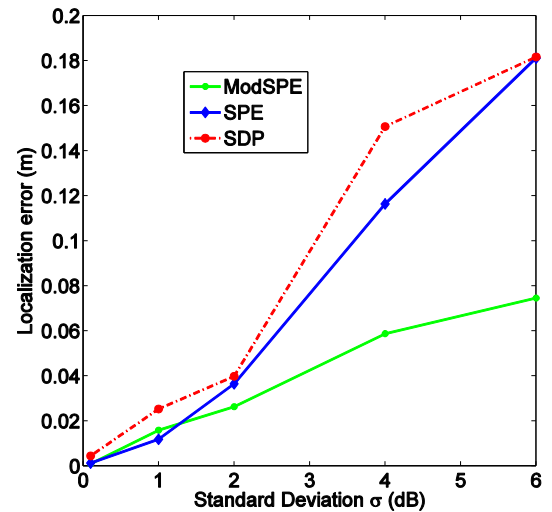


Figure 3: Localization error plotted against error standard deviation. Modified SPE parameters are Learning parameter  $\lambda = 1$ , Steps=120 and cycles= 50.

### 7.0.2 Impact of SPE Parameters

The parameters of modified SPE like learning parameter  $\lambda$ , number of cycles C and number of steps S have a significant impact on localization error. This impact is studied in detail. Here,  $\sigma$  is kept as constant at 1.

1. In Fig. 4 the number of steps in modified SPE is



varied and the localization performance is studied. After 10 steps, the error remains almost constant. Learning parameter  $\lambda = 1$  and the number of cycles is 50.

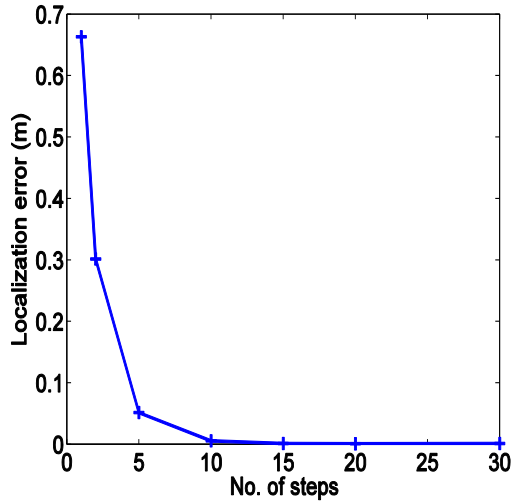


Figure 4: Study of modified SPE performance with varying number of steps.  $\sigma = 1$ . Modified SPE parameters: Learning parameter ( $\lambda$ )=1 and cycles=50.

2. In Fig. 5 localization performance is studied for varying values of cycles. As expected, localization error comes down as the number of cycles increases. However after C=40 steps, there is not much improvement. The modified SPE parameters used are: number of steps S=120 and learning parameter  $\lambda = 1$ .

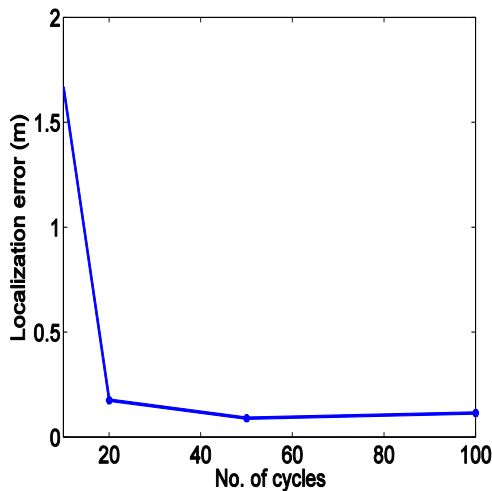


Figure 5: Study of localization performance against varying number of cycles. Here  $\sigma = 1$ . Modified SPE parameters: Learning parameter ( $\lambda$ )=1 and steps= 120.

3. The localization performance is plotted against varying  $\lambda$  in Fig. 6. Performance of the algorithm is found to be good when the initial learning parameter is around 1. When it is too large, the performance is poor due to the oscillatory nature of the solution. Also, when it is too small, the convergence is slow

leading to poor performance.

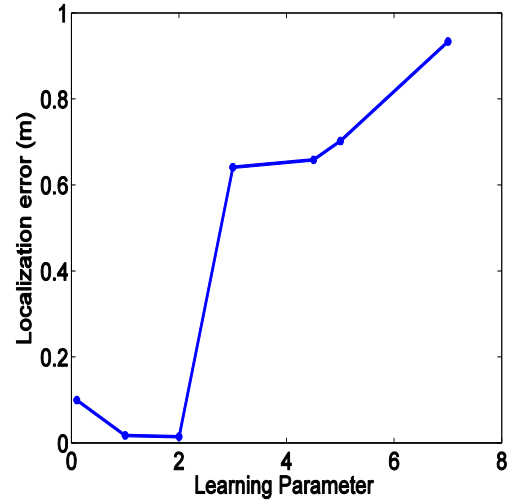


Figure 6: Performance of localization error versus initial learning parameter.  $\sigma = 1$ . Modified SPE parameters: Steps=120 and cycles= 50.

### 7.0.3 Varying number of anchor nodes

Intuitively, localization error should come down in increasing number of anchor nodes. Fig. 7 confirms this intuition, although the improvement is slight.

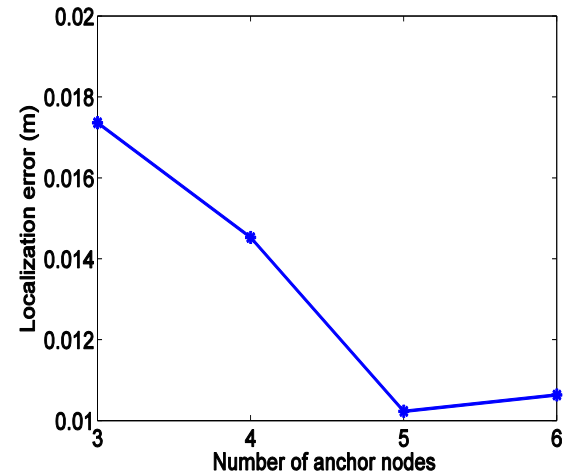


Figure 7: Localization performance against anchor node number.  $\sigma = 1$ . Modified SPE parameters: Learning parameter ( $\lambda$ )=1, steps=120 and cycles= 60.

## 8 Conclusion

A modified version of Stochastic Proximity Embedding for localization in wireless sensor networks was proposed in this paper. Correlation between measurement errors of received signal strength is taken into account in the algorithm and hence is expected to give superior performance when deployed in field. Simulation study was conducted on this algorithm and it was found to give better performance than competing algorithms like Semidefinite programming and original Stochastic Proximity Embedding.

## 9 Acknowledgment

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