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Prediction of Fatigue Life of 8090 Al-Alloy Using Beta Model

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Abstract : In the present investigation, fatigue crack growth tests of 8090 Al-alloy have been conducted using compact tension (CT) specimen under constant amplitude loading. The experimental data have been smoothened by using exponential model. A new method called 'Beta model' has been devised and proposed to predict the fatigue life of the alloy from predicted crack lengths obtained from the model results. It has been observed that the overall growth rate of the model reasonably matches with the experimental values.

Keywords: Fatigue crack growth rate; fatigue life; Beta model, stress intensity factor, maximum stress intensity factor, compact tension.

1. Introduction

Fatigue is an important mode of failure in almost all engineering components. It results mainly due to variable loading or more precisely because of cyclic variations in the applied loading or induced stresses. Hence, fatigue life prediction is important in order to enhance the residual life of the structures/components from both economic and safety point of view. Conventional life prediction procedures are generally based on the safe-life approach. In this approach, components of a structure are replaced when the probability of failure reaches a prescribed level, even though some of them may have a significant residual life. Hence, it is a highly conservative approach coupled with a penalty on economy. To avoid this, the damage-tolerant approach is often a suitable alternative for life predictions. In this approach, the fatigue specimens are undergone several coupon tests by using fracture mechanics principle until complete fracture of the specimens take place in order to know the growth rate. Once the nature of fatigue crack growth rate is known, the schedule inspection intervals are set for suitable repair/replacement of parts in order to avoid catastrophic failure. It not only minimizes the cost but can save human life as well.

However, the fatigue crack growth tests are very costly and time consuming. For example, most high strength steels take months together to complete a fatigue test. Hence, several prediction models have been suggested to predict the fatigue life of materials under various loading conditions. Among them the most primitive models are Paris model [1], Forman model [2], and Walker model [3]. However, each model has its own merits and demerits. The most important demerit in almost all deterministic models is that it involves robust numerical integration scheme in order to determine the fatigue life from crack growth rate equation. To avoid this, different soft-computing methods have been applied to predict the fatigue life. In the present work, a new method called 'Beta model' has been proposed to predict the fatigue life of 8090 Al-alloy under constant amplitude loading. It has been observed that the proposed model predicts the fatigue life with reasonable accuracy.

2. Material and Brief Experimental Procedure

In the present study, the material used for fatigue crack growth rate (FCGR) study is 8090 Al-alloy with T651 heat treated condition. The compact tension (CT) specimens with a V-starter notch have been prepared from 12.5 mm thick plates in the longitudinal transverse (LT) direction as shown in Fig. 1.

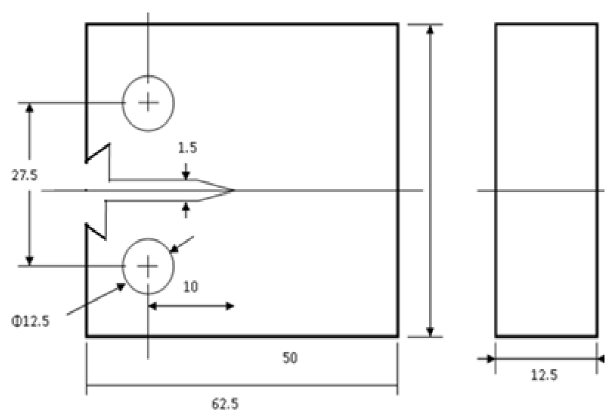

Fig. 1 – Compact tension (CT) specimen geometry

Table 1: Composition (in % wt.) of 8090 T651 Al alloy

Zn	Mg	Cu	Fe	Si	Mn	Ti	Li	Al
5.7	0.55	1.14	0.05	0.03	0.1	0.03	2.34	Bal.

Table 2: Mechanical properties of 8090 T651 Al-alloy

Young's modulus (GPa)	Yield strength (MPa)	Tensile strength (MPa)	Elongation (%)
81.2	430.0	480.0	13.0

The chemical composition and the mechanical properties of the alloy have been given in Table 1 and 2 respectively. Both the sides of the specimen surfaces have been mirror-polished by different grades of emery papers and

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subsequently by magnesium oxide (MgO) powder suspension. The specimen surfaces have been marked at 1 mm interval each and a pair of knife edges has been fixed on the face of the machined V-notch. The crack opening displacement (COD) gauge has been mounted on the knife edges to monitor the crack extension.

Initially the specimens have been fatigue pre-cracked to 1 mm in mode-I loading (crack opening mode) at constant stress intensity factor (ΔK) under given loading conditions (which include frequency: 6 Hz; load ratio: 0.1; initial and final crack lengths: 15.4 and 38.4 mm respectively). Then the constant amplitude loading (CAL) fatigue crack growth rate (FCGR) tests have been carried out under tension-tension loading in a servo-hydraulic dynamic testing machine (*INSTRON* 8502) having a load capacity of 250 kN interfaced to a computer for machine control and data acquisition. All the FCGR tests have been conducted in constant load control (increasing ΔK) mode in accordance with ASTM E647.

Experimental results

After FCGR tests, the raw laboratory data of number of cycles (N) corresponding to the specified crack lengths (each 1 mm interval marked on the specimen) have been recorded and plotted in Fig. 2. From crack length and number of cycles (a~N) plot, it has been observed that the experimental data obtained from laboratory tests contain scatter, though it is exponentially increasing in nature.

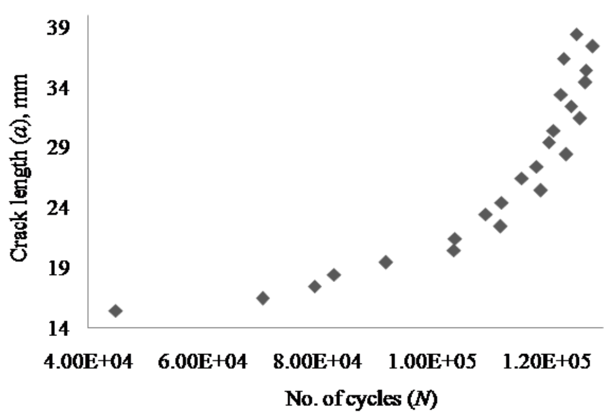

Fig. 2 – Raw experimental a~N curve

Hence, the scattered experimental a~N data have been smoothened by adopting the procedure of 'exponential model' as per author's earlier work [4]. The details of procedure have been mentioned in previous work for reader's reference. After data smoothening, the modified a~N curve has been plotted in Fig. 3 and their corresponding numerical values along with ΔK and K_{max} values have been presented in Table 3. These smoothened data has been taken as actual experimental a~N data along with their corresponding ΔK and K_{max} values for model formulation.

Table 3 – Smoothened Experimental results

No. of cycle (N)	Crack length (a) Mm	Range of stress intensity factor (ΔK)	Maximum stress intensity factor (K_{max})
44800	15.4	6.945611	7.717344
70520	16.4	7.469381	8.29931
77070	17.4	8.031633	8.924035
90955	18.4	8.637961	9.597733
97055	19.4	9.29469	10.32743
101955	20.4	10.00893	11.12103
106085	21.4	10.78861	11.98734
109095	22.4	11.64256	12.93617
112530	23.4	12.5805	13.97834
115275	24.4	13.61314	15.12571
117175	25.4	14.75216	16.39129
118785	26.4	16.01029	17.78921
120370	27.4	17.40131	19.33479
121630	28.4	18.94013	21.04459
122570	29.4	20.64278	22.93641
123365	30.4	22.52643	25.02937
123990	31.4	24.6095	27.34389
124430	32.4	26.91159	29.90176
124705	33.4	29.45358	32.72619
124890	34.4	32.25762	35.84179
125020	35.4	35.34717	39.27463
125110	36.4	38.74704	43.05226
125250	37.4	42.4834	47.20377
125325	38.4	46.58379	51.75976

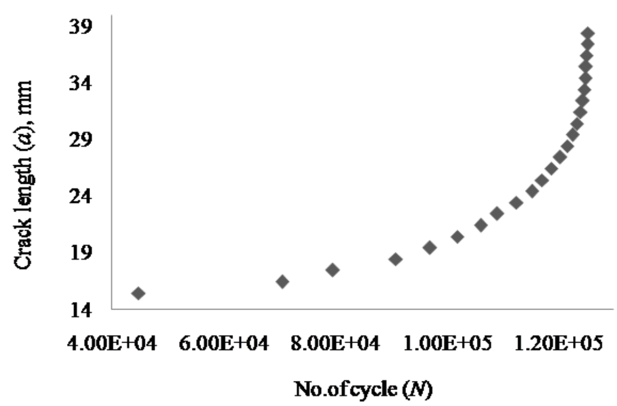

Fig. 3 – Smoothened experimental a~N curve

Formulation of Beta Model

A model is a simplified representation of a real system. There are two concepts in this definition. Firstly the system is a group of objects (components or factors) that interact with one another in an organized manner and the net result of their interaction produces the system's behavior, function and purpose. Secondly, when a system is represented or described in a simpler form known as model, the model becomes a tool to understand a system by helping to sift through its complexity and to focus on the important, relevant aspects.

The beta function was first studied by Euler and Legendre and was given its name by Jacques Binet. Just as the gamma function for integers describing factorials, the beta function can define a binomial coefficient after adjusting indices. The beta function was the first known scattering amplitude in string theory, first conjectured by Gabriele Veneziano.

The beta function can be defined in the integral form [5]

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx \quad (1)$$

Where, $a > 0$, $b > 0$; 'x' is a random variable, a and b are shape parameters. Shape of the distribution is governed by these two parameters. The silent features of beta function are as follows:

- It is flexible in describing various asymmetric sigmoid patterns.
- Its parameters are numerically stable in statistical estimation.
- Like Weibull function it predicts zero mass at time zero, but its extension to deal with various initial conditions can be easily obtained.
- Relative to the truncated expolinear equation it provides more reasonable estimates of final quantity and duration of a growth process.

The standard density function of a 'beta-distribution' is characterized by a unimodal response to an independent variable 'x' in the range of [0, 1]. The function has a density of zero when $x \leq 0$ or $x > 1$ and a maximum density at an optimum 'x' between 0 and 1. In the present fatigue crack growth rate (da/dN) problem, replacing the independent variable x with number of cycles (N) between a base number of cycles (N_b) and a maximum number of cycles (N_e) leads to an expression that can be used to describe beta growth rate.

$$\frac{da}{dN}=C_c\left\{\frac{(N_e-N)}{(N_e-N_c)}\right\}*\left(\frac{(N-N_b)}{(N_c-N_b)}\right)^{(N_c-N_b)/(N_e-N_c)^{\delta}} \quad (2)$$

Where C_c is the constant growth rate, which is achieved at number of cycle N_c. N_b and N_e are the number of cycles at the beginning and end of the growth period, respectively. If reference is set as zero (i.e. $N_b = 0$), then Eq. (1) becomes simpler. By varying the parameter δ, various curvatures in the growth rate curve for the periods $N < N_c$ and $N > N_c$ can be produced.

Growth equations are usually presented in an integral form because instantaneous growth rates are not amenable to experimental collection. The equation for the total crack length in the present context can be obtained by integrating equation (2) with respect to number of cycles. However, a definite analytical solution to the integral of equation (2) does not exist. Therefore a simplification has to be made to arrive at a definite integral growth equation. Hence, a new three parameter growth equation taking initial crack length zero has been formulated for the present situation. The integration of growth rate equation to find out crack length is given by the following equation.

$$a=a_{\max}*\left\{1+\frac{(N_e-N)}{(N_e-N_c)}\right\}*\left(\frac{N}{N_e}\right)^{\frac{N_e}{(N_e-N_c)}} \quad (3)$$

Where, $a_{\max}$ is the maximum value of crack length which is reached at N_e.

Equation (3) obeys the constraints that $a = 0$ at the start of the growth (i.e. $N = 0$) and $a = a_{\max}$ when growth is terminated (i.e. $N = N_e$). It can be applied to growth within a time span $0 \leq N \leq N_e$; otherwise 'a' has to be set as 0 if $N<0$, and $a_{\max}$ if $N > N_e$. Because Eq. (3) still produces an asymmetrical unimodal curve if N_e is exceeded. The equation with the extension that 'a' is $a_{\max}$ if $N > N_e$ is referred to as beta sigmoid growth function.

One property of beta growth function is that like Weibull equation, it always predicts the initial crack length as zero. But if the initial crack length will be considered then the beta growth function can easily be extended. Hence, Eq. 3 will be modified as:

$$a=a_b+(a_{\max}-a_b)*\left\{1+\frac{(N_e-N)}{(N_e-N_c)}\right\}*\left(\frac{N-N_b}{N_e-N_b}\right)^{\frac{(N_e-N_b)}{(N_e-N_c)}} \quad (4)$$

Next step is to introduce the non dimensional parameters such as stress intensity factor range, critical stress intensity, and maximum stress intensity factor in the above equation. These are the important parameters for fatigue crack growth and have been considered while developing the 'exponential model' for predicting fatigue life under constant amplitude loading [6]. It is essential here to mention that 'Beta model' is another approach to predict the fatigue life under constant amplitude loading for the present work. Now, the modified equation can be written as:

$$a=a_b+(a_{\max}-a_b)*\left\{1+\frac{(N_e^{\beta}-N^{\beta})}{(N_e^{\beta}-N_c^{\beta})}\right\}*\frac{(N^{\beta}-N_b^{\beta})}{(N_e^{\beta}-N_b^{\beta})}^{\frac{(N_e^{\beta}-N_b^{\beta})}{(N_e^{\beta}-N_c^{\beta})}} \quad (5)$$

The parameter β has been chosen in such a way that it becomes a non-dimensional parameter yet representing the properties that affect crack growth. Its value is approximately constant for small interval of time. Hence, β can be expressed as follows:

$$\beta=\left(\frac{\Delta K}{K_C}\right)*\left(\frac{K_{\max}}{K_C}\right) \quad (6)$$

The value of 'β' increases with increase in the value of ΔK. As value of 'β' changes with change in loading condition as well as crack length, it is needed to correlate the parameter 'β' with two crack driving forces ΔK and K_{max} [7-9] responsible for crack growth and with the material parameter such as plane stress fracture toughness (K_C). Fatigue crack growth depends on both ΔK and K_{max} in order to consider effects of mean stress. Since the modeling covers region III of FCGR curve, the value of fracture toughness (K_C) has been considered.

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Validation of results and discussion

As per the model equation (Eq. 5) and the data presented in Table-3, initial crack length (a_b) is 15.4 mm; final (i.e. maximum) crack length (a_{max}) is 38.4 mm; initial number of cycles (N_b) is 44800; and final number of cycles (N_e) is 125325 respectively. In order to predict the crack growth rate, different assumed values of number of cycles (N_c) corresponding to constant growth rate have been chosen and the corresponding crack lengths have been calculated and presented in Table 4.

Table-4 Predicted values of crack length

SL. No.	Crack length (N_c= 0.88N_e)	Crack length (N_c= 0.92N_e)	Crack length (N_c= 0.94N_e)	Crack length (N_c= 0.96N_e)	Crack length (N_c= 0.97N_e)
01	15.4	15.4	15.4	15.4	15.400
02	15.795	15.416	15.400	15.4	15.400
03	17.128	15.601	15.421	15.400	15.400
04	21.212	17.018	15.816	15.424	15.801
05	24.631	19.078	16.765	15.567	16.418
06	27.815	21.716	18.417	16.018	17.216
07	30.652	24.701	20.771	17.023	18.154
08	32.687	27.265	23.164	18.442	19.210
09	34.846	30.450	26.609	21.163	20.184
10	36.344	33.028	29.801	24.421	21.659
11	37.212	34.718	32.114	27.267	23.454
12	37.810	36.016	34.034	29.985	26.396
13	38.255	37.109	35.773	32.785	29.819
14	38.489	37.796	36.954	34.932	32.740
15	38.585	38.175	37.661	36.363	34.861
16	38.607	38.388	38.106	37.365	36.460
17	38.583	38.477	38.337	37.960	37.479
18	38.542	38.493	38.427	38.247	38.009
19	38.507	38.467	38.450	38.360	38.239
20	38.479	38.451	38.451	38.405	38.343
21	38.457	38.438	38.443	38.420	38.389
22	38.441	38.414	38.434	38.423	38.407
23	38.415	38.406	38.414	38.412	38.410

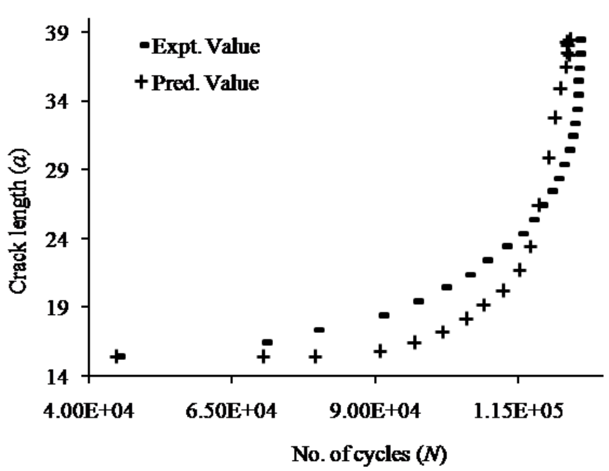

Fig. 4 – Comparison of experimental and predicted a~N curve

The crack growth rate (da/dN) has been calculated by taking the predicted crack length (a) corresponding to $N_c = 0.97 \times N_e$ (last column of Table-4) and smoothened values of number of cycles (N) by using the following formula.

$$\frac{da}{dN} = \frac{a_2 - a_1}{N_2 - N_1} \qquad (7)$$

The curves of crack growth rates (da/dN) with stress intensity ranges (ΔK) have been plotted in Fig. 5 for both predicted and experimental values.

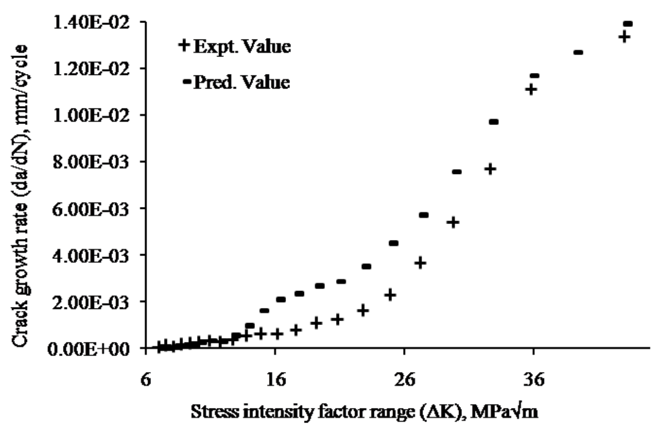

Fig. 5 – Comparison of da/dN~ΔK curve

From the above plots it has been observed that at about the middle of the growth period (i.e. at a = 26.4 mm and N = 118785) the predicted and experimental values almost matches with percentage of deviation of 0.015. Before that the model slightly overestimates the fatigue life whereas after that it underestimates the life. However, observing the overall growth period from initial to final values the prediction results reasonably matches with the experimental values.

Conclusion

In the present work, fatigue crack growth tests of 8090 Al-alloy have been conducted under constant amplitude loading condition. The raw experimental data of crack lengths and number of cycles have been smoothened by using exponential model to minimize the scatter. A numerical model called ‘Beta model’ has been applied to predict the crack lengths at different number of cycles corresponding constant growth rate (N_c). It has been observed that the predicted results at $N_c = 0.97 \times N_e$ gives best results in comparison to experimental values with reasonable accuracy.

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