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Variable Structure H∞ Controller for Aircraft

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Abstract: This paper proposes a new synthesis method and tuning procedure for a variable structure H∞ multivariable controller for fixed wing aircrafts attitude control, which consists in switching from one-degree of freedom controller (1-DOF) to two degrees of freedom controller (2-DOF), and vice versa, with a method for bump-less transfer in switching transitions . The proposed method is tested using robustness analysis based on frequency domain and time analysis computer simulations based on a nonlinear mathematical model of the F-16 aircraft published by the NASA, from which satisfactory results are obtained.

Keywords: Robust control, H∞ controller, variable structure control, bump-less transfer control.

I. Introduction

This paper deals with the controller design and parameters tuning problem for multivariable control of a fixed wing aircraft, using a variable structure H∞ control strategy. In practice, conventional methods (based mainly on combination of PID regulators) are habitually used [i, ii] in flight control systems (FCS) and in industrial processes control. There are many publications over H∞ control theory, for example in [iii-ix] and references therein, to name just a few; which can be complemented with specific software aimed to assist engineers during the controller design [x-xiii]. Nevertheless, the main problem consists on an suitable selection of the weighting transfer functions, which not only gives a mathematical solution of the problem, but also a practical algorithm implementable in a digital processor for hard real time control.

In this paper we present a novel method for designing multivariable H∞ control for aircrafts, based on a variable structure using the mixed sensitivity problem [iii-viii], where an suitable selection of the weighting transfer functions is proposed for obtaining a numerical solution and a realizable implementation for hard real time control. In order to characterize controller robustness and performance, a nonlinear mathematical model of a F-16 aircraft (published by the NASA [ii, xiv, xv]) is used in the hardware in the loop simulations (HILS) carried out .

The rest of this paper is organized as follows: Section 2 deals with the statement of the problem to be solved and the variable structure controller design method proposed in this paper is described. Controller design for F-16 aircraft, robustness and performance analysis are carried out in Section 3, and finally, conclusions are drawn in Section 4.

II. H∞ controller design problem

The H∞ problem consists in to find a controller, Gc, which stabilizes the closed-loop system, T_{zw}, given in Figure 1, where the input-output relation between input vector, w, and the output vector, z, is given by

$$z = T_{zw}\, w,$$

where the restriction $\|T_{zw}\|_\infty, < \gamma$, is satisfied for a real number $\gamma > 0$. In this work the following mixed sensitivity H_∞ problem is considered [iii-viii],

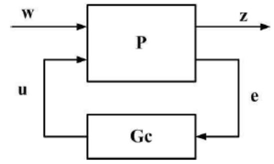
$$T_{zw} = [\ W_S S_o \quad W_R S_o G_c \quad W_T T_o\ ]^T,$$

where the input vector is given by w = r (set-point), and the output vector, $z = [z_1\ \ z_2\ \ z_3]^T$, is compound by three vectors: $z_1 = W_S e$, where the error vector is e = r - y; $z_2 = W_R u$, where u is the controller output vector; and $z_3 = W_T y$, where *y* is the process variable or controlled variable vector. The weighting transfer functions, $\{W_S(s)\ ,\ W_R(s),\ W_T(s)\}$, depend on frequency (where "s" is the Laplace variable), and can be used for performance and stability robustness specifications expressed in the frequency domain. T_o is named as the complementary output sensitivity function,

$$T_o(s) = G(s)\, G_c(s)\, [\ I + G(s)\, G_c(s)\ ]^{-1},$$

where G is the plant mathematical model used for controller (G_c) design and S_o is the output sensitivity function, where S_o and T_o are related by $S_o + T_o = I$, where I is a compatible unit matrix; and $S_o G_c$ is named as the control sensitivity function. Through an appropriate choice of these weighting transfer functions, specifications related with closed-loop system performance and robustness qualities can be considered a priori for controller design, such as set-point tracking errors, disturbance rejection, system bandwidth, control effort, and stability robustness with respect to some types of uncertainties in the plant model used for controller design [iii-x].

In this paper we propose a procedure for designing and fine-tuning the H_∞ controller for a fixed wing aircraft. Taking into account the space limitations, we focus on a particular controller design for lateral dynamics, specifically, the coordinated turn maneuver, but the procedure is applicable to different controllers that form part of a flight control system.



$$\begin{aligned} \dot{x}_p &= A_p x_p + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ e &= C_2 x + D_{21} w + D_{22} u \end{aligned}$$

Figure 1. Generalized plant (P) matrices and controller (Gc).

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In Figure, 1 the generalized plant (P) matrices and controller (Gc) used in the standard H control problem [iii, iv, v] are shown.

In this paper, we propose two control structures, the first one, or Control Structure 1 (CS1), uses a one Degree-Of-Freedom H∞ Controller (1-DOF-HC) named Gc1; and the second one, or Control Structure 2 (CS2), uses a two Degrees-Of-Freedom H∞ Controller (2-DOF-HC), composed with Gc1 (controller used in CS1) and Gc2, where Gc2 is designed using as process to control the closed-loop system composed with the plant to be controlled (aircraft mathematical model used for Gc1 controller design) and controller Gc1 (see Figure 2).

The following steps are followed for Gc1 design (CS1):

Step1. For a given flight condition, a linear time invariant (LTI) model is used, $G_H \equiv (A_{GH},B_{GH},C_{GH},D_{GH})$. A tuning parameter, τ_H, is obtained taking into account the dynamics of the aircraft. For the lateral dynamics, the value of τ_H is obtained from the eigenvalue associated to the roll convergence mode, λ_ϕ, using the following approximations,

a) $\tau_H \approx 1/|\lambda_\phi|$, or in alternative form, b) $\tau_H \approx 1/(2|\lambda_\phi|)$

For example, in case of a F-16 aircraft with λ_ϕ = -3.62, the first approximation is employed, with τ_H = 0.25 s; meanwhile for a Boeing-747 with λ_ϕ = -0.56, the second approximation is used, with τ_H = 1 s.

Step 2. Weighting transfer functions $\{W_S,W_R,W_T\}$ are calculated depending on τ_H and two tuning parameters $\{\rho_H, \beta_H\}$, using tuning formulae obtained experimentally as an adaptation (taking into account aircraft dynamics) of the methods given in [xvi-xviii]:

$$W_S(s) = K_S((s+pS)/(s+pS_1))(pS_1/pS),$$
$$pS = \rho_H (1/\tau_H),\ K_S=10^{-5},\ pS_1=2\ 10^5\ pS,$$
$$W_R(s) = \beta_H\ , \qquad W_T(s) = 0.8$$

Step 3. Based on model (G_H) matrices $(A_{GH},B_{GH},C_{GH},D_{GH})$ and on $\{W_S,W_R,W_T\}$, the generalized plant model, P_1, is obtained, which consists of nine matrices (se Fig. 1):

$$P_1 \equiv (A_{p1}, [B_1, B_2]_1, [C_1, C_2]_1, [D_{11}, D_{12}\ ;\ D_{21}, D_{22}]_1).$$

Step 4. The 1-DOF-HC (Internal H_∞ controller, Gc1, is obtained using published algorithms [x-xiii]. This controller achieves that the resultant closed loop system, T_{z1w1}, be asymptotically stable and the relation between z_1 and w_1 satisfies $\|T_{z1w1}\|_\infty < \gamma_1$, where γ_1 is minimized using an iterative procedure with a resolution value of 0.01 as stop condition for the computation algorithm. As initial value is used $\gamma_1 = 1$.

Step 5. Discrete time controller version of G_{c1} is obtained using bilinear transformation for a selected sampling time, T_m = 0.001 seconds.

Once Gc1 has been obtained for CS1, we pass to Gc2 design for CS2. The following steps are followed:

Step 6. External H∞ controller, Gc2, is obtained using the closed-loop system composed with the plant to be controlled (aircraft mathematical model used for Gc1 controller design) and controller Gc1. We propose to employ the same weighting transfer functions as in step 3; and the corresponding generalized plant model, P_2, is obtained, which consists of new nine matrices:

$$P_2 \equiv (A_{p2}, [B_1, B_2]_2, [C_1, C_2]_2, [D_{11}, D_{12}\ ;\ D_{21}, D_{22}]_2).$$

For this generalized plant, the controller, Gc2, achieves that the resultant closed loop system, T_{z2w2}, be asymptotically stable and $\|T_{z2w2}\|_\infty < \gamma_2$. where γ_2 is minimized using an iterative procedure with a resolution value of 0.01 as stop condition for the computation algorithm. As initial value is used $\gamma_2 = 1$.

Step 7. Discrete time controller version for Gc2 is obtained using bilinear transformation for a selected sampling time, T_m = 0.001 seconds.

The resultant discrete time algorithms for Gc1 and Gc2 are implemented in EPESC [xix], a hardware/software environment for HILS (Hardware In The Loop Simulation), where it is tested the proposed control strategy experimental realizability in hard real time.

Variable structure controller

Based on experimental results observed in simulations with SC1 and SC2, a variable structure H∞ controller is proposed. This controller combines the favorable properties of both controllers in order to satisfied the following objectives: 1) To improve disturbance rejection for disturbances acting at the plant input, 2) to increase the robustness with respect to uncertainties located as plant feedback, 3) to increase robustness with respect to aircraft non-linear dynamics, 4) to improve tracking properties for set-point changes for maneuvers that require big changes in set-point and produce high nonlinear behavior in aircraft dynamics and actuators. Variable structure control supposes a switching from SC1 to SC2, or vice versa, when an event condition (EC) is satisfied (see Figure 2). For initial condition state (S0 in Figure 3), it is supposed that the aircraft is flying straight and level. In this case, the CS2 is active, since the objective is to hold flying condition. When a manoeuvre is needed and a set-point change is commanded (Event Condition 1, EC1), CS1 must be activated switching from CS2 to CS1 (a time counter is set to zero, $t_{SP} = 0$), and the control system will be in state 1 (S1, in Figure 3) during a time period T_{SW} (in simulations we have used T_{SW} = 0.7 s). Once $t_{SP} = T_{SW}$, the Event Condition 2 (EC2) is satisfied and the control structure changes from CS1 to CS2, which corresponds to state 2 (S2) . The control system will stay in state S2 until new event condition (EC1) happens. In this case, a state transition from S2 to S1 will be done and the procedure will be repeated as it is shown in Figure 3.

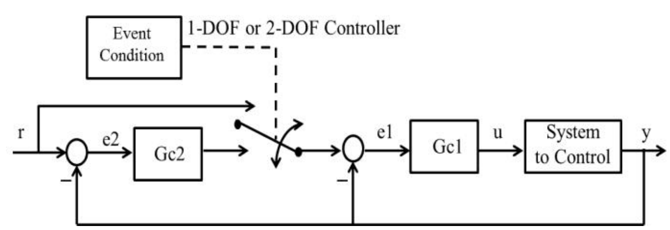

Figure 2. Variable structure H_∞ controller proposed in this paper, where an event condition determines the switching between control structures.

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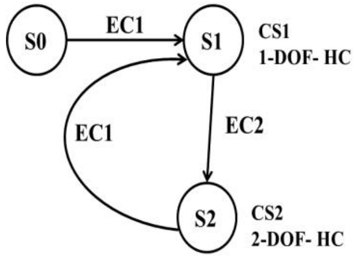

Figure 3. States (S0, S1, S2) diagram with Event Conditions (E1, E2) for Variable Structure H_∞ controller.

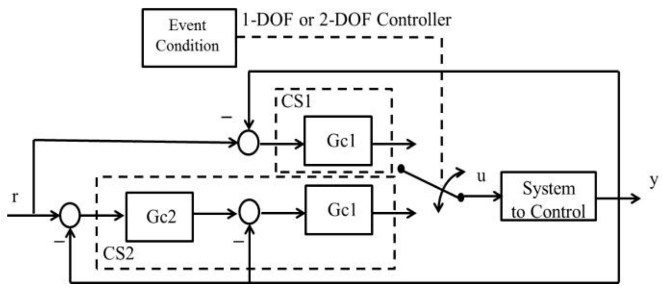

Figure 4. Variable structure H_∞ controller (VSHC) realization used for BLT controller design and implementation.

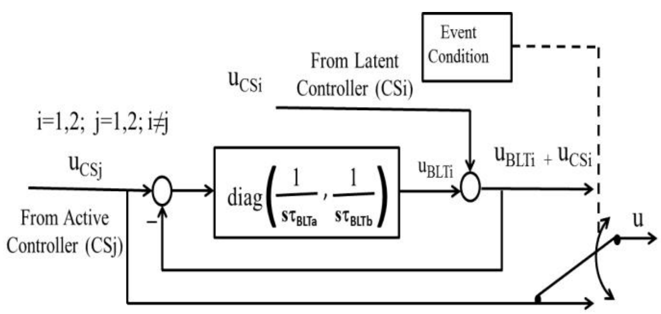

Figure 5. BLT controller for switching from SCi to SCj.

In order to avoid bump effect in switching from CS1 to CS2 and vice versa, the respective bump-less transfer controllers (BLTC1 and BLTC2) must be designed for CS1 and CS2. In order to carry out the design and implementation of the BLT controllers, the structure given in Figure 4 must be used. In Figure 5, it is shown the BLT for CSj (BLTCj) when CSi is the active controller and CSj is the latent controller (i=1,2; j=1,2; i≠j). It can be observed that the obtained closed-loop that relates u_{CSj} with $(u_{BLT}+u_{CSi})$ is given by $1/(\tau_{BLT}\ s+1)$, where τ_{BLT} is a specified time constant. Due to controller output vector has two components, two time constants ($\tau_{BLTa} = \tau_{BLTb}$) must be specified. In simulations we have used $\tau_{BLTa} = \tau_{BLTb} = 0.1$ s.

III. Controller design for F-16 aircraft

The controller design method proposed in previous Section is applied to control the lateral dynamics of a F-16 aircraft. The objective is to provide coordinated turns by causing the bank angle ϕ to follow a set-point, while maintaining the sideslip angle β at zero. It is a two-channel system with two process variables (PV), ϕ and β, and two manipulated variables (MV): ailerons angle deflection δ_a and rudder angle deflection δ_d. The reference vector or set-point is SP = $[\phi_{SP}\ \beta_{SP}]^T$. To obtain an useful mathematical model for controller design, the nonlinear F-16 model [xiv,xv] was linearized for a nominal flight condition (H =1000 m, velocity V_T = 153 m/s) considering that the aircraft is flying straight and level, and using as state variables: sideslip (β), bank angle (ϕ), roll rate (p) and yaw rate (r); for which the following state matrices (A,B,C,D) are obtained,

$$A=\begin{pmatrix} -0.27226 & 0.064005 & 0.046908 & -0.99182 \\ 0 & 0 & 1 & 0.047038 \\ -28.201 & 0 & -3.2574 & 0.63476 \\ 7.2278 & 0 & -0.033499 & -0.45418 \end{pmatrix}$$

$$B=\begin{pmatrix} 0.00023708 & 0.00071060 \\ 0 & 0 \\ -0.62218 & 0.11520 \\ -0.026767 & -0.058442 \end{pmatrix}$$

$$C=\begin{pmatrix} 0 & 180/\pi & 0 & 0 \\ 180/\pi & 0 & 0 & 0 \end{pmatrix}, \quad D=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

where the International System of Units is used for state variables, while degrees are used for controlled or process variables (PV) , PV = $[\phi\ \ \beta]^T$.

For obtaining (A_{GH},B_{GH},A_{GH},B_{GH}) matrices (Step 1), additional states (δ_a and δ_r) are introduced by the aileron and rudder actuators; each of them is modeled as a first order transfer function with time constant (1/20.2) seconds [ii, xiv, xv]. Rudder has mechanical limitations given by [-30,30] deg., and [-120,120] deg/s; and for ailerons limitations are [-21.5, 21.5] deg. and [-80, 80] deg/s. These restrictions must be taken into account during the controller design and analysis procedures in order to obtain a realizable digital control law implementable in a HILS environment.

Following the steps given in previous Section, once τ_H and ρ_H are fixed, τ_H = 0.25, ρ_H = 1, the unique tuning parameter is β_H, which is used as fine tuning parameter. In Figure 6, time responses for β_H = 0.03, 0.01 and 0.003 are shown for unit step change [1 0] degrees in set-point (applied at t = 0.2 s) using a first order low pass filter with time constant t_f = 0.1 s for set-point; and an unit step change, [1 -1] degrees, for simulating equivalent disturbance acting at the plant input (applied at t = 2 s). Although β_H (used as fine tuning parameter once τ_H and ρ_H have been fixed) affects to system response for SP change (tracking problem), its main effect is over disturbance rejection, as it can be seen in Figure 6. The criteria used for β_H fine tuning has been to achieve satisfactory responses for large changes in SP (tests for 40 and 80 degrees have been carried out), where the nonlinear dynamics of aircraft and actuators are made evident. Finally, β_H = 0.01 has been used for the VSHC design.

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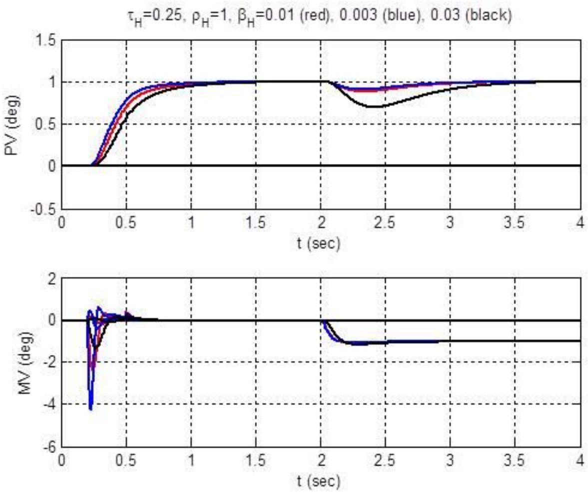

Figure 6. VSHC time responses for β_H = 0.03, 0.01 0.003.

Control system robustness can be studied in an analytical way using robustness indicators based on frequency (ω, rad/s) domain. Some robustness indicators take into account un-modeled dynamics in the plant model used for controller design, considering multiplicative equivalent uncertainties [iii-viii]. In this paper we have considered the following conservative robustness indicators, which consider unstructured uncertainties and are based on the employ of singular values:

1) Multiplicative Stability Margin for equivalent multiplicative uncertainty located at the plant input (MSMi):

$$MSM_i = \frac{1}{\sup_\omega \overline{\sigma}[T_i(j\omega)]}\,100\%$$

2) Multiplicative Stability Margin for equivalent multiplicative uncertainty located at the plant output (MSMo):

$$MSM_o = \frac{1}{\sup_\omega \overline{\sigma}[T_o(j\omega)]}\,100\%$$

3) Plant Feedback Stability Margin for equivalent uncertainty located as plant feedback (PFSM):

$$PFSM = \frac{1}{\sup_\omega \overline{\sigma}[S_o(j\omega)G(j\omega)]}\,100\%$$

In case of partial knowledge of uncertainty structure, robustness indicators based on structured singular values could be used [iii-x], nevertheless, we have consider the most conservative case of unstructured uncertainty and robustness indicators based on singular values. In Table 1, robustness indicators MSMi, MSMo, PFSM) are given for 1-DOF H∞ controller (1-DOF-HC) and for 2-DOF H∞ controller (2-DOF-HC). As it can be seen, the worst case indicators for MSMi and MSMo are greater than 60 %, although it is observed that 1-DOF-HC has better values for these indicators.

Table 1. Robustness indicators.

Control Structure	MSMi	MSMo	PFSM
1-DOF	100 %	100 %	50 %
2-DOF	63.8 %	78.3 %	380 %

Nevertheless, the main difference between both controllers is the PFSM indicator, due to in case of 1-DOF-HC the value is 0.5 and for the 2-DOF-HC the value of 3.8 is obtained. This is significant fact, taking into account that the greater the PFSM indicator is, the better properties for input disturbance rejection and robustness with respect to nonlinear dynamics of the aircraft are obtained. For that, the use of the 2-DOF controller is justified. In simulation tests, it has been observed that a combination of both controllers implemented in a variable structure controller gives better results than the case of using 1-DOF-HC (better for SP changes and with higher MSMi and MSMo) or 2-DOF-HC (better for rejection of disturbances acting at the plant input and with higher PFSM) independently, such as it has been proposed in Section II.

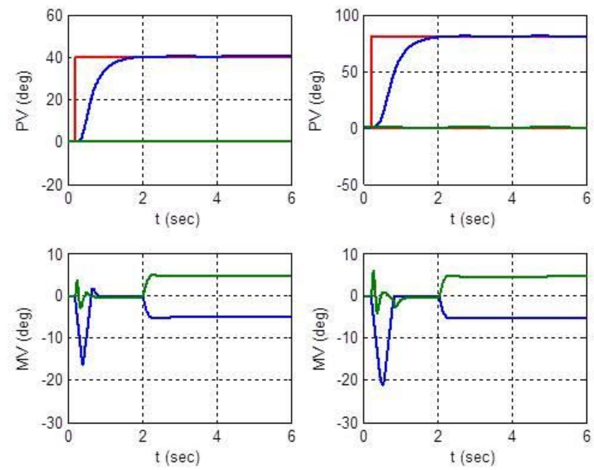

Figure 7. VSHC time responses for V_T = 153 m/s.

In Figure 7, the time responses are obtained for step changes (applied at t = 0.2 s) of [40 0] degrees and [80 0] degrees respectively, and for a disturbance (equivalent to [5 -5] degrees) acting at plant input (applied at t = 2 s). For simulation tests it is used a discrete time version of the variable structure H∞ controller proposed in this paper (sampling frequency 1000 Hz) and the nonlinear model of the F-16 aircraft [xiv, xv]; which have been implemented in EPESC [xix], a Hardware In the Loop Simulation (HILS) environment. Due to the large values of SP, the actuator limitations and the high nonlinear aircraft dynamics are made manifest in a significant way; nevertheless, simulation tests probe the VSHC robustness is sufficient for obtaining satisfactory behavior for flying conditions very different to nominal condition. A robustness test of the VSHC (which has been designed for a nominal velocity of 153 m/s) consists in applying a fixed controller for different flying velocities as it is shown in Figures 8-9, where satisfactory time responses are obtained for velocities 180 m/s and 250 m/s, which supposes velocity variations with respect to nominal value (153 m/s) of 17.6 % and 63.4 % respectively. Taking into account the simulations and analysis carried out, the properties of the VSHC are the following: 1) overshoot lower than 2% and rise time ≤ 1 s for extreme changes in roll angle, with very reduced interaction over sideslip angle, 2) zero stationary error for step changes in set-point (tracking problem), and for fixed set-point (regulation problem) when step changes in disturbance acting at the plant input are applied (which demonstrates the robustness of the VSHC with respect to significant bias errors, or equivalent faults, in actuators), which avoids the problem of the mixed sensitivity design problem described in [vii], 3) a realizable implementation in hard real time environment is obtained for the VSHC digital version.

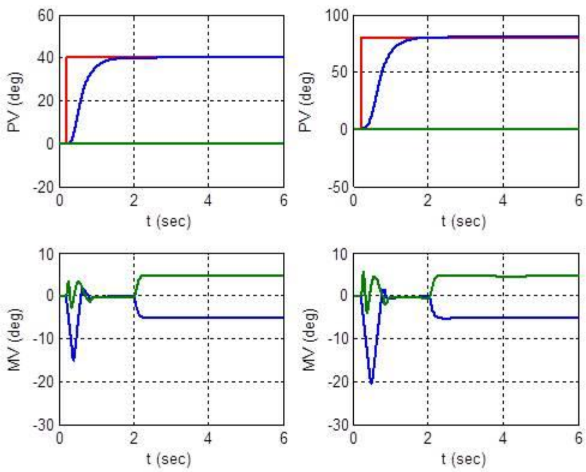

Figure 8. VSHC time responses for V_T = 180 m/s.

IV. Conclusions

A variable structure H∞ controller (VSHC) and a method for tuning weighting transfer functions for using in flight control systems have been proposed in this paper. Only two parameters are used for controller tuning, one of them (τ_H) associated to aircraft dynamics and the other one (ρ_H) used for adjusting time response characteristics and robustness indicators based on frequency domain. Satisfactory simulation results have been obtained with the nonlinear model of F-16 aircraft (developed by the NASA) for large coordinate turn manoeuvres which demand large roll angles and reveals the high non-linear dynamics of the F-16 aircraft.

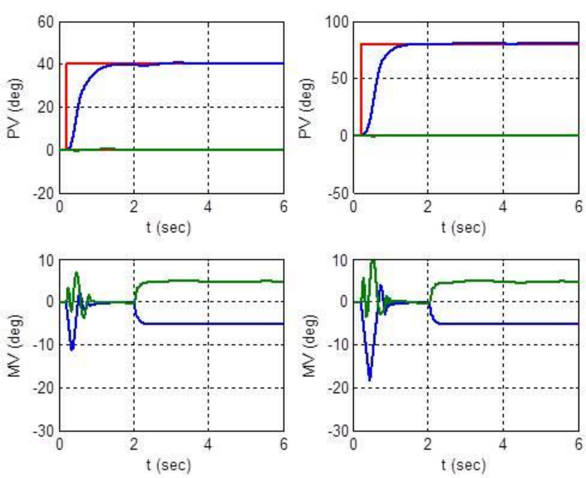

Figure 9. VSHC time responses for V_T = 250 m/s.

References

i. *K. J. Astrom, T. Hägglund (2006). PID Controllers: Theory, Design and Tuning. Instrument Society of America (ISA).*

ii. *B.L. Stevens, F.L. Lewis (2003), Aircraft Control and Simulation, Willey.*

iii. *K. Zhou, J. C.. Doyle, K. Glover (1997). Robust and Optimal Control. Prentice Hall.*

iv. *O. H. Bosgra, H. Kwakernaak, G. Meinsma (2008). Design Methods for Control Systems. Dutch Institute of Systems and Control (DISC).*

v. *Skogestad, S., I. Postlethwaite (2002), Multivariable Feedback Control, Wiley.*

vi. *J.M. Maciejowski (1989). Multivariable Feedback Design. Adison Wesley.*

vii. *H. Kwakernaak (2002). Mixed Sensitivity Design: An Aerospace Case Study. IFAC World Conference 2002, 5pp.*

viii. *M.J. Grimble (2001), Industrial control systems design. Wiley.*

ix. *D. C. Donha, M. R. Katebi (2007). Automatic weight selection for Hinf controller synthesis. International Journal of Systems Science. Vol. 38, No. 8, pp 651-664.*

x. *G. Balas, R. Chiang, A. Packard, M. Safonov (2011). Robust Control Toolbox for Matlab. Mathworks.*

xi. *G. Balas, J. C. Doyle, K. Glover, A. Packard, and R. Smith (1999), Analysis and Synthesis Toolbox, Musyn Inc. and The MathWorks Inc.*

xii. *P.M. Thompson (2001). Program CC, Tutorial and User's Guide. Systems Technology, Inc., UK.*

xiii. *D-W- Gu, P.H. Petkov, M. M. Lonstantinov (1999). H-inf and H-2 Optimization Toolbox in SLICOT. Europepan Community BRITE-EURAM III Thematic Netwowrks Programme NICONET.*

xiv. *Nguyen, L.T., Ogburn, M.E., Gilbert, W.P., Kibler, K.S., Brown, P.W., and Deal, P.L. (1979), "Simulator Study of Stall / Post-Stall Characteristics of a Fighter Airplane With Relaxed Longitudinal Static Stability", NASA TP 1538.*

xv. *L. Sonneveldt (2010). Nonlinear F-16 Model Description. Control & Simulation Division. Faculty of Aerospace Engineering. Delft University of Technology. The Netherlands.*

xvi. *M. J. López, J. Terrón, H. Haro (1997). H-infinity design practice methodology for process control. Proceedings IASTED International Conference on Control, pp 199-202. Cancum (Mexico).*

xvii. *Manuel J. López (1999). Automatic Tuning of H∞-infinity Controllers for Processes Control, Cadiz University. Spain.*

xviii. *M. J. López, A. Consegliere (2001). A method for tuning H∞ controllers.*

xix. *Proceedings European Control Conference ECC'2001, pp 1559-1564.*

xx. *L. García, M. J. López, J. Lorenzo (2006), Hard Real Time Based on*

xxi. *Linux/RTAI for Plant Simulation and Control Systems Evaluation, WSEAS Transactions on Systems and Control, Issue 2, Vol. 1, pp 161*-168.