

Variable Step Size Normalized Least Mean Square Algorithm for Mobile Communication

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Abstract — *This paper describes a robust variable step size normalized least mean square algorithm (VSSNLMS) to enhance interference suppression in smart antenna system. The fixed step size normalized least mean square (NLMS) will result in a trade-off issue between convergence rate and steady-state mean square error (MSE) of NLMS algorithm. The purpose of a variable step-size normalized LMS algorithm is to solve the dilemma of fast convergence rate and low MSE. The proposed VSSNLMS algorithm reduces the MSE and shows faster convergence rate when compared to the conventional LMS and NLMS. Moreover, the enhanced performance of the new VSSNLMS algorithm is validated from the output beam pattern.*

Keywords— Beamforming, LMS, MSE, NLMS, Smart antenna, VSSNLMS

I. INTRODUCTION

Smart Antenna is recognized as an important technique for increasing the user capacity of a wireless communication network, this is due to its capabilities of steering nulls to reduce co-channel interference and pointing independent beams toward various users as well as its ability to provide estimates of directions of radiating sources. The core of Smart Antenna is the selection of smart adaptive beam forming algorithm. An adaptive algorithm is said to be “blind” if it does not require a known training sequence. Antennas are usually Smart antennas used to provide significant advantages and improved performance in almost all wireless communications systems. These Smart antennas dynamically adapt to changing traffic requirements. Smart employed at the base stations and they radiate narrow beams to serve different users [1-3].

Smart antenna system consists of antennas and the associated digital control system which provide the “intelligent” beam-forming hence the term smart antenna. A smart antenna which is held in the base station of a mobile system comprises of a uniform linear array (ULA) antenna where the amplitudes are accustomed by a group of complex weights using an adaptive beamforming algorithm. Before adaptive beamforming, direction of arrival estimation is used to specify the main directions of users and interferers [4-5]. Many adaptive beamforming algorithms are based on the Least Mean Square (LMS). The normalized LMS (NLMS) performance is better than the conventional LMS. This is due to the fact that NLMS uses a variable step-size parameter. The variation in the step-size is accomplished because of the division process at each iteration of the fixed step size by the input power [6-7].

This paper demonstrates that the combination between beamforming and VSSNLMS leads to better results than NLMS and all other LMS-based algorithms (LMS and VSS-LMS) in terms of interference suppression [10].

II. ADAPTIVE BEAMFORMING

A. LMS Algorithm

Least mean squares (LMS) algorithms are class of adaptive filter used to mimic a desired filter by finding the filter coefficients that relate to producing the least mean squares of the error signal (difference between the desired and the actual signal). The algorithm starts by assuming a small weights (zero in most cases), and at each step, by finding the gradient of the mean square error, the weights are updated. The LMS algorithm is the most widely used adaptive beamforming algorithm, being employed in several communication applications. It has gained popularity due to its low computational complexity and proven robustness. The LMS algorithm changes the weight vector $w(n)$ along the direction of the estimated gradient based on the steepest descent method. In employing the LMS algorithm, it is assumed that sufficient knowledge of the reference signal is present.

Derivation of LMS weight vector

The weight update equation of Steepest decent method is given by

$$w(n+1) = w(n) + \mu E \{ e(n) x^*(n) \} \quad (1)$$

Where, E is the expectation operator, μ is the step size used for convergence of beamforming algorithm, $e(n)$ is the error signal and $x^*(n)$ is the conjugate of induced signal.

A practical limitation with steepest decent method is that $E \{ e(n) x^*(n) \}$ is generally unknown. Therefore, it must be replaced with an estimate such as sample mean given by

$$E \{ e(n) x^*(n) \} = \frac{1}{L} \sum_{l=0}^{L-1} e(n-l) x^*(n-l) \quad (2)$$

Incorporating this estimate as in equation (2) into steepest decent algorithm, the update for $w(n)$ is given by

$$w(n+1) = w(n) + \mu \frac{1}{L} \sum_{l=0}^{L-1} e(n-l) x^*(n-l) \quad (3)$$

A special case of equation (3) occurs if we use a one point sample mean as given by

$$E\{e(n)x^*(n)\} = e(n)x^*(n) \quad (4)$$

Using the condition of equation (4), the weight vector update equation assumes a particular simple form as given by

$$w(n+1) = w(n) + \mu e(n) x^*(n) \quad (5)$$

Where, μ is the step size which can be in the range given by

$$0 \leq \mu \leq \frac{2}{3tr(R_{xx})} \quad (6)$$

Where, $tr(R_{xx})$ is the trace of auto correlation matrix.

For simulation, step size is given by

$$\mu = \frac{2}{3tr(R_{xx})} \quad (7)$$

The advantage of using LMS algorithm over steepest decent method is that the expectation operator in steepest decent method is removed from weight vector by considering a single sample.

B. NLMS Algorithm

The main drawback of the "pure" LMS algorithm is that it is sensitive to the scaling of its input. This makes it very hard to choose a step size μ that guarantees stability of the algorithm. The Normalized least mean squares (NLMS) algorithm [8], [9] is a variant of the LMS algorithm that solves this problem by normalizing with the power of the input.

The weight for the NLMS algorithm is given by

$$w(n+1) = w(n) + \frac{\mu e(n) x(n)}{\|x(n)\|^2} \quad (8)$$

Sometimes $x(n)$ which is the input signal becomes very small which may cause $w(n+1)$ to be unbounded. However, to avoid this situation; sigma which is a constant value is added to the denominator which made the NLMS algorithm be described as

$$w(n+1) = w(n) + \frac{\mu e(n) x(n)}{\sigma + \|x(n)\|^2} \quad (9)$$

$\mu = \text{step size}$

$e(n) = \text{error signal}$

$x(n) = \text{recieved signal}$

$\sigma = \text{sigma}$

$w(n) = \text{weight}$

The step size is given by

$$\mu = \frac{2}{3tr(R_{xx})} \quad (10)$$

Where R_{xx} is the autocorrelation matrix.

B. Variable Step Size Normalized Least Mean Square (VSSNLMS)

The main goal of the developed Variable Step Size (VSS) NLMS algorithm is to replace the fixed step size μ that is used in conventional NLMS by a variable one. This is to avoid a trade-off issue between convergence rate and steady-state MSE. In this algorithm a large step size is used in the initial stages to speed the rate of convergence and a smaller step size is used near to the steady state of the Mean Square Error (MSE) to obtain an optimum value. The array weight is given by

$$w(n+1) = w(n) + \frac{\mu(n) e(n) x(n)}{\sigma + \|x(n)\|^2} \quad (11)$$

And the step size varied for the various iterations and is given by the equation

$$\mu(n) = \left\{ \left(\frac{6}{N} \right)^2 \left(k \left(\frac{6}{N} \right)^2 \right) + 0.001 \right\}, 1 \leq k \leq \frac{N}{6}$$

$$0.0001 \qquad \qquad \qquad \frac{N}{6} \leq k \leq N$$

$N = \text{number of iterations}$

The steps of LMS and NLMS are similar as that of proposed VSSNLMS algorithm but step size calculation is different. The steps of VSSNLMS algorithm are summarized below. Step: 1 Compute the steering vector for desired direction θ_0 .

$$a(\theta_0) = \begin{bmatrix} 1 \\ e^{-j\pi \sin \theta_0} \\ \vdots \\ e^{-j\pi(L-1) \sin \theta_0} \end{bmatrix} \quad (12)$$

Step: 2 Compute the array manifold vector corresponding to M interference source directions

$$a(\theta_1), a(\theta_2), \dots, a(\theta_M)$$

Step: 3 Obtain signal samples 'S' by sampling continuous time signal of baseband frequency. (For simulation sine wave samples is considered).

Step: 4 compute the autocorrelation matrix R_{xx} .

Step: 5 compute the step size by using equation

$$\mu = \frac{2}{3tr(R_{xx})}$$

Step: 6 Compute the following for all signal samples $0 \leq n \leq N_s$. Where, N_s is the total number of signal samples.

$$x(n) = a(\theta_0) s(n) + i(n) \sum_{i=1}^M a(\theta_i) + n_0(n)$$

$$y(n) = w(n)^T x(n)$$

$$e(n) = s(n) - y(n)$$

$$w(n+1) = w(n) + \frac{\mu(n)e(n)x(n)}{\sigma + \|x(n)\|^2} \quad \text{where}$$

$$\mu(n) = \begin{cases} \left(\frac{6}{N}\right)^2 \left(k - \left(\frac{N}{6}\right)\right)^2 + 0.001 & 1 \leq k \leq \frac{N}{6} \\ 0.0001 & \frac{N}{6} \leq k \leq N \end{cases}$$

N = number of iterations

Step: 7 The array factor is computed

$$AF = \sum_{i=1}^L w^H(i) e^{j\pi i \sin \theta} \quad (13)$$

Step: 8 Array factor versus angles are plotted.

III. SIMULATIONS

The LMS, NLMS and proposed VSSNLMS algorithm are simulated using MATLAB software. The parameters used in the simulations are tabulated in Table I. This paper shows computation of mean square error and beam pattern.

A. Comparison of MSE for LMS, NLMS and Proposed VSSNLMS

In VSSNLMS algorithm, the value of the step size changes with time, while in standard LMS and NLMS algorithm the value of the step size is fixed. Fig. 1, Fig. 2, Fig. 3 shows beam patterns of LMS, NLMS and VSSNLMS algorithms respectively. Comparison of LMS, NLMS, and VSSNLMS algorithms with respect to MSE v/s number of iterations is shown in Figure 4. By observing Fig. 4, the convergence is faster when the adaptive VSSNLMS algorithm is used rather than LMS and NLMS algorithm.

TABLE I
 PARAMETERS USED IN SIMULATIONS

Sl no:	Parameters	Values
1	Type of antenna array	Uniform Linear Array
2	Number of array elements	Variable (8, 10, 30, 70, 100)
3	Pass band Frequency range	(3-4) GHz
4	Voltage range for AOA	(1-5)v
5	Direction range for AOA	0 to $\pm 90^\circ$
6	Simulation Language	MATLAB

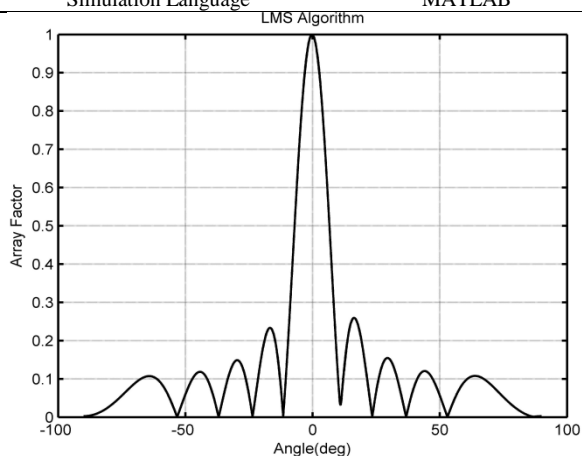


Fig.1: LMS Algorithm Radiation Pattern

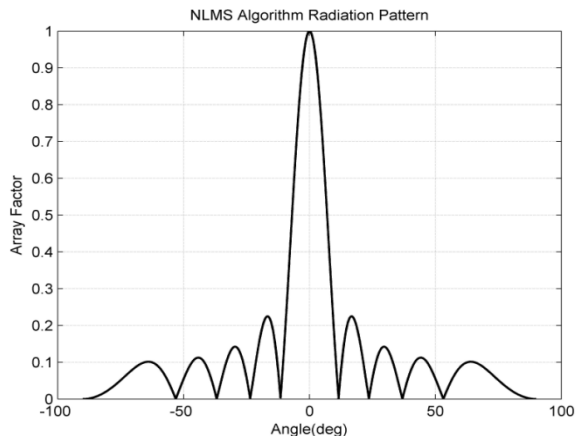


Fig.2: NLMS Algorithm Radiation Pattern

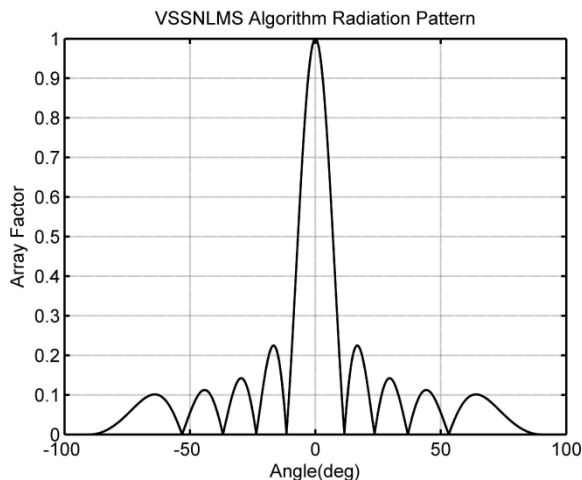


Fig.3: VSSNLMS Algorithm Radiation Pattern

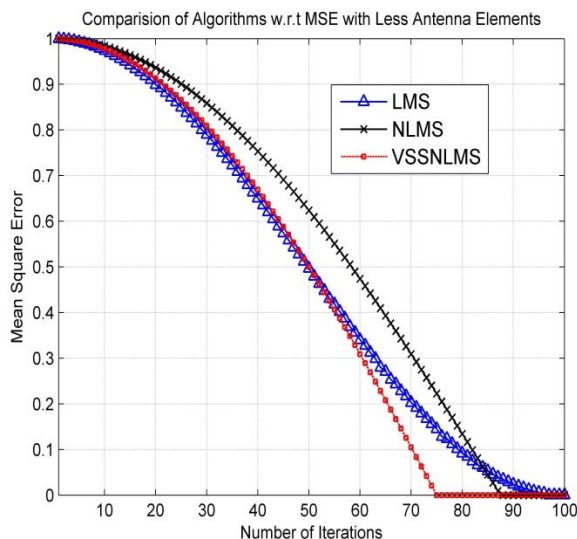


Fig.4: Comparison of algorithm w.r.t mean square error v/s no. of iterations.

Comparison of beam pattern shown in Figure 5 for the LMS, NLMS and proposed VSSNLMS, respectively, it is clearly shown that the proposed VSSNLMS outperforms LMS and NLMS.

V. CONCLUSION

The paper introduced and examined the use of normalized least mean square and new variable step size normalized least mean square VSSNLMS algorithm in the design of adaptive beamforming smart antenna system for interference suppression. The used VSSNLMS algorithm results in faster convergence rate when compared to NLMS and LMS algorithm. The mean square error (MSE) obtained from VSSNLMS algorithm was low as compared to conventional NLMS and LMS. The robust performance of the proposed VSSNLMS algorithm takes the form of better interference suppression. Hence Increases the performance of the smart antenna.

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