

Multi-Channel Queuing Modeling on System Delivery Service : A Case Study of Banque Populaire Du Rwanda Kimironko Branch

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Abstract: *The long waiting time in banking hall may cause dissatisfaction to the customers; therefore the quit of the customers, almost all banks in Rwanda exercise multi-channel queuing system to provide services. In this research we use the multichannel queuing model to measure the service performances of Banque Populaire du Rwanda Kimironko branch which is taken as case study of this research. The researcher provides the necessary information to the bank managers on what can be done to reduce the long waiting time of the customers. Through observation data have been collected in ten days which are the peak days of the month i.e. the end and the start of the month. The research also analyzes the waiting lines of Banque Populaire du Rwanda by compare the incoming customer of the first 30 minutes of every peak day of the month. The study have found that the average arrival customers (λ) is 128 customers per hour and the average service rate of (μ) is 21 customers, The utilization rate of the system is 1.527 which means that the tellers are busy 152.7% of the time, on 30th and 30th of the month are the days which rank first among other peak days of the month only in 30 first minutes the banking hall has almost 45 customers waiting for services. To reduce the long waiting of the customer in Bank, the number of servers should be increased especially in the peak days of the month*

Key words: Queue, Multichannel, waiting time, arrival customer, service rate, utilization rate.

I. Introduction

For many decades, it has been a complex decision environment to improve the performance of service

delivery where arrival and service time are randomly distributed and performed by human employee (Ullah et.al, 2014[1]; Azmat [2]. The banking industry can be considered as a good example. Queuing theory is the study of waiting lines which is a common feature in organizations that provide services where customers arrive at a random distribution to receive service; it is a part of operation research since the results found are often used when making business decisions about the resources needed to provide a service. Queuing models is suitable to be applied in the banking system, because it is helpful in modeling the queue management of the bank.

According to Eshetie, [3] Banks as queue system has been studied by various scholars with different scenarios and channels such as single server (figure 1), multiple servers (figure2), parallel server, different queue discipline, as well as various arrival and service time distributions. The multi channel queuing model has been a topic for different researches in various domains. Olaniyi [4] observed that there was a preference to use multi-channel queue system as compared to single queue in banks due to cost and customer satisfaction implications. Odunukwe [5] said that single-channel queuing system with multiple servers (i.e. $M|M:k$ system where $k > 1$) are operated by banks with the hope of giving customers maximum satisfaction and also make their profit, the advantage of this model is that the slowness of one server does not affect the movement of the queue because every customer has a choice to go to the next free server instead of waiting to the slow server. Since queues are present in a lot of service areas, therefore multichannel queuing model has been used not only in the banking industry but also in various industries such as health care centers, transportations systems, grocery checkout, call center etc by Albert Attakora [6]

In Bank the characteristics of arrival rate is considered to be infinite since the arrival rate does not depend on the number of customers being served; the arrival of customers are random it follows Poisson distribution. Most Bank system use queue discipline known as First come, First Served (FCFS) also called FIFO (First In, First Out). This is not applicable in all services especially in emergencies. Many Banks of today use multi-channel service system since there is more than one teller and each customer is on single common line waiting for the first free teller. Depending on the teller capacity, the customers are not served in the same amount of time, thus the service time is random and it is assumed to be exponentially distributed. The long waiting time in banking hall should be well managed; otherwise it can cause the dissatisfaction of customers. Though the BPR is a multi channel system in providing the teller's services, its service operation does not consider the multi channel queue model to optimize the number of servers. The application of the model can be the solutions to these challenges in analyzing the queuing system of Banque Populaire du Rwanda at Kimironko Branch and provide the performances to the bank management for improvements of service delivery.

II. Material and Methodology

In this research, the customers come from infinite population and the system is able to serve all the customers come for teller's services. The service is occupied by more than one teller. The number of the tellers of BPR Kimironko Branch is Seven. As it is more than one the fitted model is: M/M/S: FCFS/ ∞/∞

Where; M stands for Markovian process, which means that the arrival is following the Poisson distribution and the service time follows the exponential distribution. S stands for multi-server, where S is equal 4.

FCFS: First Come, First Served which is the followed discipline

∞ : infinite source of arrival

∞ : infinite service limit

Our sample is going to use non-probability sampling. For our case the sample size is 10 days, which are the peak days of the month i.e. the start and the closure week of the month. This will take six working hours, from 10:00 AM to 1:00PM and from 02:00PM to 05:00 PM. In this study. Interpretation will be carried out by using of descriptive statistics and significance test.

Let us assume that the customers are from single line and then waiting for first available teller, each of these channels has an independent and identical exponential service time distribution with the mean $1/\mu$ and the arrival process has exponential distribution with rate λ . Arrival will join single queue then after enter the first available channel. The multi channel here follows the discipline of First Come, First served. Balking, reneging and Jockeying will not be assumed. The following are the performance parameters of multi channel queuing model with Poisson arrival and Exponential service Times (M/M/N).

$$1. \text{ System utilization } \rho = \frac{\lambda}{s\mu} \quad (1)$$

Where λ is the mean arrival rate of the customers (expected number of arrivals per unit time); s is the number of the servers (tellers), in our model S is equal to four; μ Is the mean service rate (expected number of customers served per unit of time)

$c\mu$ is the total service rate. Service utilization factor is the proportion of time on average that each server is busy; the total service rate $c\mu$ must be greater than the arrival rate λ i.e. $c\mu > \lambda$, otherwise if $c\mu < \lambda$ the queue will grow without bound (Bannerman, [7])

$$\frac{\lambda}{c\mu} < 1 \quad (2)$$

Let assume that $\lambda_n = \lambda$ and $\mu_n = n\mu$

$$\mu_n = \begin{cases} n\mu & \text{when } n < c \\ c\mu & \text{when } n \geq c \end{cases} \quad n \text{ is the number of units}$$

Since everyone who comes should join the queue so we have $P_0, P_1, P_2, \dots, P_n$

$P_n = \rho^n P_0$ Which is the general expression of Probability P_n of n units in the system P_n

$$= \frac{\lambda^n}{\mu^n} P_0 \quad (3)$$

Since μ change with n and c, then P_n will be written as

$$P_n = \frac{\lambda^n}{\mu \cdot 2\mu \cdot 3\mu \dots n\mu} \quad \text{When } n < c \quad (4)$$

$$= \left(\frac{\lambda}{\mu}\right)^n \cdot \frac{1}{n!} \cdot P_0$$

Otherwise when $n \geq c$

$$P_n = \frac{\lambda^n}{\mu \cdot 2\mu \cdot 3\mu \cdot 4\mu \dots c\mu} \cdot P_0 = \frac{\lambda^n}{c! \mu^n c^{n-c}} P_0 \quad (5)$$

For normalization equation $\sum_{n=0}^{\infty} P_n = 1$, then we have $P_0 + P_1 + P_2 + \dots + P_{\infty} = 1$, the general expression of P_n is

$$\sum_{n=0}^{c-1} \frac{\lambda^n}{n!} P_0 + \sum_{n=c}^{\infty} \frac{\lambda^n}{c! c^{n-c}} P_0 = 1 \quad \text{It implies that}$$

$$P_0 \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \sum_{n=c}^{\infty} \frac{\rho^c}{c! c^{n-c}} \right] = 1$$

$$P_0 \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left\{ 1 + \frac{\rho}{c} + \left(\frac{\rho}{c}\right)^2 + \dots \infty \right\} \right] = 1 \quad (6)$$

this is the infinite geometric series.

as $\frac{\rho}{c} = \frac{\lambda}{c\mu} < 1$, We use summation formulae for infinite geometric series.

$$P_0 \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left(\frac{1}{1-\frac{\rho}{c}} \right) \right] = 1$$

$$\text{Finally } P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left(\frac{1}{1-\frac{\rho}{c}} \right)} \quad (7)$$

This is the probability that there zero units/customer in the system.

For multi-channel system, it is customary to define $L_q = \sum_{n=c}^{\infty} (n-c) P_n$, $q=0$, when $n=c$

By substitution of P_n , if $j = n - c$

$$L_q = \sum_{j=0}^{\infty} j P_{c+j} \quad , \text{ we use the 2}^{nd} \text{ condition which s}$$

$$n \geq c \Rightarrow L_q = \sum_{j=0}^{\infty} j P_{c+j} = \sum_{j=0}^{\infty} j \frac{\rho^{c+j}}{c!c^j} P_0 =$$

$$\sum_{j=0}^{\infty} \frac{\rho^{c+1} P_0 j \rho^{j-1}}{c!c^{j-1}} \quad (8)$$

$$= \frac{\rho^{c+1}}{c!c} P_0 \sum_{j=0}^{\infty} \frac{d(\rho/c)^j}{d(\rho/c)}$$

$$= \frac{\rho^{c+1} P_0 d \Sigma(\rho/c)^j}{c!c d(\rho/c)}$$

The last term is the infinite geometric series

$$= \frac{\rho^{c+1} P_0}{c!c} \frac{d}{d(\rho/c)} \frac{1}{1 - (\rho/c)}$$

As we know $\rho/c < 1$

$$\text{Then } L_q = \frac{\rho^{c+1}}{c!c} P_0 \frac{1}{(1-\rho/c)^2}$$

Finally

$$L_q = \frac{\rho^{c+1} P_0}{(c-1)!(c-\rho)^2} \quad (9)$$

Having calculated L_q by the formulae (9), we can compute W_q by using the little's law formula

$$L = \lambda w, \text{ therefore } W_q = \frac{L_q}{\lambda} \quad (10)$$

Where,

W_q is the average time a customer wait on the queue

Average number of units in the system

$$L = L_q + \frac{\lambda}{\mu} = \frac{\lambda \mu \left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c-\rho)^2} P_0 + \frac{\lambda}{\mu} \quad (11)$$

Average time a unit spends in the system

$$W = W_q + \frac{1}{\mu}$$

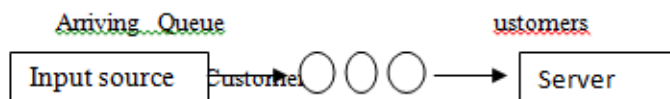


Figure 1: Structure of single queuing system of the bank with single server (M/M/1)

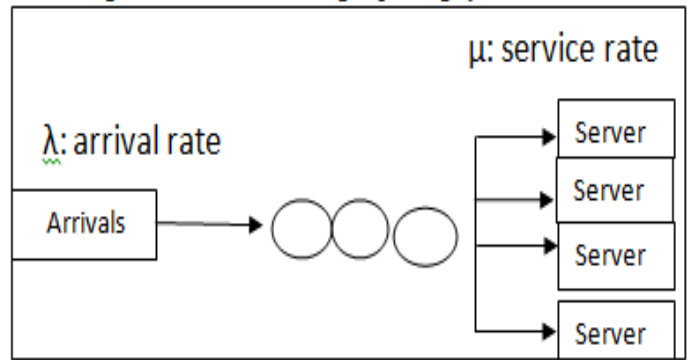


Figure 2: Multi-channel system of the bank (M/M/Cesults and Tables

The data was collected in Banque Populaire du Rwanda-Kimironko Branch; data have been collected in 10 days which are considered as peak days of the month i.e. the start and the end days of the month. The collection was based on the number of customers who come for teller's services i.e. to withdraw or to deposit money hence this represents customer's arrival time. Finally after Calculations, a research has found that in ten days, Banque Populaire du Rwanda- Kimironko has the overall average of arrivals $\lambda=128.3$ customers arrived per hour. (Table 1)

The researcher found that the highest average is 29.83 customers per hour on 30th and 27.16 customers per hour on 29th of the month.

On the 29th and 30th of the month, the bank increase the number of tellers to 5, because those days are known to have the crowd of customers because they are the last days of taxes payment. The overall average number of customers served per hour $\mu=21.46$ i.e. **21 customers** served per hour.

During the collection of the data the researcher found that customers entered the banking hall frequently on specific dates and less frequently on some other dates within the peak days of the month. The researcher took a sample of first 30 minutes, the below graph depicts sample of the arrival time of both peak days was plotted with number of customers against arrival time. It shows a graph of the number of customers arriving at a banking hall against their respective arrival time. It is visible on the graph that 29th and 30th of the month are the days which rank first among other peak days of the month only in 30 first minutes the banking hall has almost 45 customers waiting for services, this show the high frequency of the customers on the two last days which is caused by the deadline of the tax payment

Incoming Customers

	N	Minimum	Maximum	Mean	Variance
Number of arrival customer per hour	10	87.33	192.67	1.2834E2	1.572E3
Valid N (listwise)	10				

Table 1: Primary data of mean number of arrivals per hour in SPSS

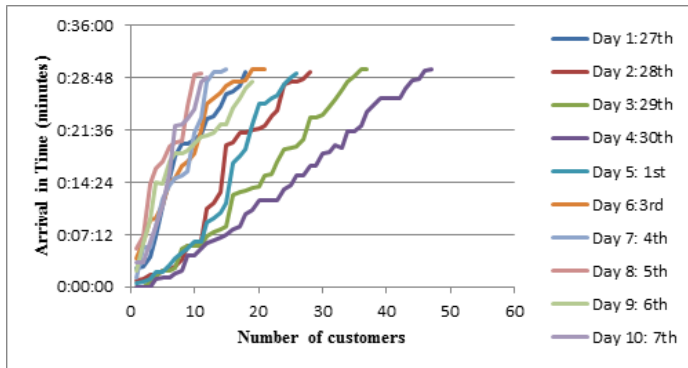


Figure 3: Arrival customers in first 30 minutes

III. Conclusion

The main objective of this study was to use multi channel queuing model to the system delivery service of Banque Populaire du Rwanda Kimironko branch.

The findings showed that the average incoming customers are 128 customers per hour and 21 customers are served per hour. This is obviously that the queue will grow without bound, in other words every customer will risk to not get the service. It is determined that a customer spends a lot of time in the banking hall of BPR Kimironko branch during the peak days of the month, and the customer waits 1.67 hour in the system to get the service. The results also showed that on 29th and 30th, these are the days which have a great number of customers in the banking hall, which are known to be last days of paying different taxes. To reduce the long waiting of the customer in Bank, the number of servers should be increased especially in the peak days of the month. This is proved that replacing four tellers by five tellers the waiting time will decrease from **0.57** to **0.09**.

IV. References

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